Some Useful Limits

The following limits can be computed using l’Hospital’s rule, but you don’t need to know l’Hospital’s rule to use them.

If $a$ and $p$ are positive constants, then

(i) $\lim_{x \to \infty} \frac{x^p}{e^{ax}} = 0$

(ii) $\lim_{x \to \infty} \frac{\ln x}{x^p} = 0$

(iii) $\lim_{x \to 0^+} x^p \ln x = 0$

Informal justifications:

(i) As $x \to \infty$, $e^{ax} \to \infty$ faster than $x^p \to \infty$

(ii) As $x \to \infty$, $x^p \to \infty$ faster than $\ln x \to \infty$

(iii) As $x \to 0^+$, $x^p \to 0$ faster than $\ln x \to -\infty$

See Section 4.4 of your textbook if you want to learn about l’Hospital’s rule.