Mathematics 101, Section 212 (CSP)
Practice Midterm Test 2
March 2011

Time: 50 minutes. This is a closed book examination: no books, notes, electronic memory or communication devices are allowed. **CALCULATORS AND CELL PHONES ARE NOT ALLOWED.**

1. **Short-Answer Questions.** Put your answers in the boxes provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. At most one mark will be given for an incorrect answer. Unless otherwise stated, **simplify your answers as much as possible in this question.**

   **Marks**

   (a) Find $M_x$, the moment about the $x$-axis, for the region between the curve $y = 2(1 - x)$ and the $x$-axis, $0 \leq x \leq 1$. For density take $\rho = 1$.

   **Answer**

   (b) Find $\lim_{n\to\infty} \tan^{-1} \left( \frac{(n-1)!}{(n+1)!} \right)$, or else determine that the sequence diverges.

   **Answer**

   (c) Determine whether the series $5 + 3 + \frac{9}{5} + \frac{27}{25} + \frac{81}{125} + \ldots$ converges or diverges. If it is convergent, find the sum.

   **Answer**
Full-Solution Problems. In questions 2-5, justify your answers and show all your work. If a box is provided, write your final answer there. If you need more space, use the back of the previous page. Unless otherwise indicated, simplification of answers is not required.

2. Evaluate the following integrals. **SHOW ALL STEPS.** No marks will be awarded for correct answers with no work shown.

(a) \[ \int x \cos 5x \, dx. \]

(b) \[ \int \sqrt{3 - 2x - x^2} \, dx. \]
(c) $\int \frac{4x - 4}{x^4 + 4x^2} \, dx.$

3. Determine whether the integral

$$\int_0^\infty \frac{1}{2e^x - x} \, dx$$

is convergent or divergent. (Remember to give justification.)
4. A vat with 1000 gallons of beer initially contains 4% alcohol by volume. Then beer with 6% alcohol is pumped into the vat at a rate of 10 gallons per minute, and the well-stirred mixture is pumped out at the same rate. What is the percentage of alcohol in the vat after 100 minutes?

5. A semicircular plate with radius 2 m is placed vertically with its straight edge along the bottom of a tank of water that is 4 m deep. Calculate the hydrostatic force on one side of the plate.