**Business Problems**

\( p = \text{unit price}. \)

\( q = \text{quantity demanded}. \) It is the number of units that will sell at price \( p. \)

We typically assume that the number of units produced is exactly the number of units that will sell. Therefore we also have:

\( q = \text{quantity produced}. \)

Sometimes we treat \( q \) as a function of \( p. \) Other times we treat \( p \) as a function of \( q. \)

An equation relating \( p \) and \( q \) is called a demand equation. The curve describing the relationship between \( p \) and \( q \) is called the demand curve.

\[
\text{Profit} = P = R - C \\
\text{Revenue} = R = pq. \\
\text{Cost} \ C \ is \ the \ sum \ of \ fixed \ cost \ FC \ and \ variable \ cost \ VC, \ i.e., \ \\
C = FC + VC. \\
\text{Variable costs} \ VC \ and \ hence \ costs \ C \ will \ be \ a \ function \ of \ price \ p \ or q, \ depending \ on \ which \ we \ regard \ as \ the \ independent \ variable. \]

We usually want to maximize profit or maximize revenue or minimize cost.

**Example.**

Epple Inc. is the only manufacturer of the popular oPad. Epple estimates that when the price of the oPad is $200, then the weekly demand is 5000 units. For every $1 increase in the price, the weekly demand decreases by 50 units. Assume that the fixed costs of production on a weekly basis are $100,000, and the variable costs of production are $75 per unit.

(a) Find the linear demand equation for the oPad with \( p \) as a function of \( q. \)

(b) Find the weekly cost function as a function of \( q. \)

(c) Find the weekly revenue as a function of \( q. \)

(d) Find the weekly profit as a function of \( q. \)

(e) What demand maximizes the weekly profit?
(a) The demand equation has the form

\[ p = mq + b. \]

The change in \( p \) per change in \( q \) is

\[ m = \frac{\Delta p}{\Delta q} = \frac{1}{-50} = -\frac{1}{50}. \]

We need to calculate \( b \).

The point \((p, q) = (200, 5000)\) is on the line.

\[ b = p - mq = 200 - \frac{1}{50} \cdot 5000 = 300 \]

\[ p = -\frac{1}{50} q + 300. \]

(b) \( C = FC + VC = 100,000 + 75q. \)

(c) \( R = pq = \left(-\frac{1}{50} q + 300\right) q = -\frac{1}{50} q^2 + 300q. \)

(d)

\[ P = R - C \]
\[ = -\frac{1}{50} q^2 + 300q - (100,000 + 75q) \]
\[ = -\frac{1}{50} q^2 + 225q - 100,000 \]

(e) The maximum occurs at the vertex of the parabola \( P(q) = -\frac{1}{50} q^2 + 225q - 100,000. \)

We could find the vertex by completing the square or finding the two zeroes and computing their average. Instead, let’s use calculus (because that’s what we will use for more complicated problems).

Set \( \frac{d}{dq} P = 0 \) and solve for \( q. \)

\[ 0 = \frac{d}{dq} P \left(-\frac{1}{50} q^2 + 225q - 100,000\right) = -\frac{1}{25} q + 225 \]

\[ q = 25 \cdot 225 = 5625 \]