Assignment 1 - Math 105 - Section 211
Hand in your solutions on Friday, January 25 at the beginning of class.

1. (a) Draw the $y = 0$ and $y = 1$ traces of the surface $z = x^2 + 2y^2$.
(b) Which of the following represents the surface $z = x^2 + 2y^2$? Justify your answer using the traces you drew in (a).

![Traces of the surface](image)

2. Let $f(x, y) = \cos(xy) \sin(x + y)$. In this exercise, you will compute the tangent plane to the graph of $f$ (i.e., to the surface $z = f(x, y)$) at the point $(\pi, 0, f(\pi, 0))$.
(a) Evaluate $f$ at $(\pi, 0)$.
(b) Compute the gradient of $f$ at $(\pi, 0)$.
(c) Find an equation of the plane through the point $(\pi, 0, 0)$ and orthogonal to the vector $\langle -1, -1, 1 \rangle$.

3. Show that the planes $x + 2y + 3z = 0$ and $-x + 5y - 3z = 0$ are orthogonal. Then find a plane through the origin that is orthogonal to both planes.

4. Find the domain and range of $f(x, y) = \frac{1}{x^2 + y^2}$. Describe the domain.

5. Suppose $f(x, y)$ has continuous partial derivatives of all orders and $f_{xx} = 3x^5 + 2\cos(x^4 + y^4)$. Find $f_{xyxy}$ and justify your answer.

6. Find all critical points of the function $f(x, y) = (x^2 + y^2)e^{x^2 - y^2}$. Classify each point as a local maximum, local minimum, or saddle point.

7. Find the absolute maximum and minimum values of $f(x, y) = 3 + xy - x - 2y$ on the closed triangular region with vertices $(1, 0)$, $(5, 0)$, $(1, 4)$.

8. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = xy + x + y$ subject to $xy = 4$.

9. A certain T-shirt company makes two types of shirts. Type A says “Winter is Coming”, while type B says “This is truly the darkest timeline”. It sells each type A shirt for $15. It sells each Type B shirt for $10. The cost of producing $x$ shirts of type A is

$$\frac{1}{200}x^2 + 6x + 1000.$$ 

The cost of producing $y$ shirts of type B is

$$\frac{1}{200}y^2 + 4y + 3000.$$ 

Determine how many shirts of each type the company should produce to maximize its profit.