Absolute Maxima and Absolute Minima

f has an absolute maximum at \((a, b)\) if 
\[ f(a, b) \geq f(x, y) \]
for all points \((x, y)\) in the domain of \(f\). The point \((a, b)\) is then called an absolute maximum point of \(f\) and the number \(f(a, b)\) is called an absolute maximum value of \(f\).

f has an absolute minimum at \((a, b)\) if 
\[ f(a, b) \leq f(x, y) \]
for all points \((x, y)\) in the domain of \(f\). The point \((a, b)\) is then called an absolute minimum point of \(f\) and the number \(f(a, b)\) is called an absolute minimum value of \(f\).

We will consider the problem of finding the absolute maximum and minimum values of \(f\) on a closed bounded set.
A set in $\mathbb{R}^2$ is called **closed** if it contains all its boundary points.

A set in $\mathbb{R}^2$ is called **bounded** if it is contained in some disk, that is, if you can draw a circle (possibly a large one) around the whole thing. Bounded sets are finite in extent.

\[
\{(x, y) : x^2 + y^2 \leq 1\} \\
\text{Closed, Bounded}
\]

\[
\{(x, y) : -4 \leq x \leq -2, 1 \leq y \leq 3\} \\
\text{Closed, Bounded}
\]

\[
\{(x, y) : x^2 + y^2 < 1\} \\
\text{Not Closed, Bounded}
\]

\[
\{(x, y) : x^2 + y^2 \leq 1, (x, y) \neq (0, 0)\} \\
\text{Not Closed, Bounded}
\]
\{(x,y) : -4 < x \leq -2, 1 \leq y \leq 3 \}\nNot Closed, Bounded

\{(x,y) : x \geq 2 \}\nClosed, Not Bounded

\{(x,y) : x^2 + y^2 > 1 \}\nNot Closed, Not Bounded