Surfaces and Traces

Ex. Use traces to graph the surface defined by the equation

\[ \frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1 \]

Roughly, a trace is a 2-dimensional slice of a surface. A trace of a surface is the set of points at which the surface intersects a plane of the form

\[ x = x_0 \quad \text{or} \quad y = y_0 \quad \text{or} \quad z = z_0 \]

(that is, a plane parallel to one of the coordinate planes)

\[ z = 0 \quad \text{trace (xy - trace)} \]

Set \( z = 0 \)

\[ \frac{x^2}{4} + \frac{y^2}{9} = 1 \quad \text{ellipse} \]

\[ z = \pm 2 \quad \text{traces} \]

Set \( z = \pm 2 \), \( \therefore z^2 = 4 \)

\[ \frac{x^2}{4} + \frac{y^2}{9} - 4 = 1 \]

\[ \therefore \frac{x^2}{4} + \frac{y^2}{9} = 5 \quad \text{ellipse (larger)} \]
\[ z = z_0 \quad \text{trace} \]

Set \( z = z_0 \)

\[
\frac{x^2}{4} + \frac{y^2}{9} = 1 + \frac{z^2}{\text{constant}}
\]

Ellipse (larger as \( |z_0| \) gets larger)

\[ x = 0 \quad (yz \text{-trace}) \]

Set \( x = 0 \)

\[
\frac{y^2}{9} - z^2 = 1
\]

Hyperbola

\[
\frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1 - \frac{x_0^2}{4}
\]

Hyperbola

\[ 1 - \frac{x_0^2}{4} > 0 \]

\[ 1 - \frac{x_0^2}{4} < 0 \]
\[ y = 0 \quad \text{trace} \quad (x^2 - z^2 = 1) \quad \text{hyperbola} \]

\[ \frac{x^2}{4} - z^2 = 1 - \frac{y_0^2}{q} \quad \text{hyperbola} \]

graph of

\[ \frac{x^2}{4} + \frac{y^2}{q} - z^2 = 1 \]