1. Let $C_{36}$ be the cyclic group of order 36. Let $a$ generate $G$, i.e. $G = \langle a \rangle$, and let $e$ denote the identity element in $G$.
(a) Find all $x \in G$ such that $x^3 = e$.
(b) Find all $x \in G$ such that $x^4 = e$.
(c) Find all $x \in G$ such that $x^5 = e$.
(d) Find all $x \in G$ such that $x^{36} = e$.

2. Find the order of the following elements of the symmetric group $S_5$:
(a) $(1\ 3)(2\ 4)$;
(b) $(3\ 5)$;
(c) $(2\ 3\ 4)(1\ 5)$.

3. For any group $G$, we let $Z(G)$ (called the center of $G$) denote the set of all $a \in G$ such that $ax = xa$ for all $x \in G$.
(a) Let $G = S_3$. Find $Z(G)$.
(b) Let $G = D_4$. Let $r$ be the element of 4 that generates the rotations in $G$ and let $f$ (having order 2) be a flip. Recall that $frf = r^3$. Find $Z(G)$.

4. Given a group $G$ and an integer $n$ let $G_n$ denote the set $\{x^n \mid x \in G\}$.
(a) Let $G$ be the symmetric group $S_3$. Write down all the elements in $G_3$. Is $G_3$ a subgroup of $G$?
(b) Let $G$ be symmetric group $S_3$. Write down all the elements in $G_2$. Is $G_2$ a subgroup of $G$?

5. Let $G$ be a group with $|G| = n$. Show that $x^n = e$ for every $x \in G$.

6. Let $G$ be a finite group and let $H$ be a subgroup of $G$ such that $|G| = 2|H|$.
(a) Show that for any $a \in G$ that is not an element of $H$, we have $a^2 \in H$. [Hint: It suffices to show that $a^2 H \neq aH$.]
(b) Show that for any $a \in G$ that is not an element of $H$, the order of $a$ is even. [Hint: Show that $a^n \neq e$ for any odd integer $n$.]