1. In (a)-(d) below, $C_m$ denotes the cyclic group of size $m$. For each question, answer yes or no and explain your answer.
(a) Is $C_3 \times C_3$ isomorphic to $C_9$?
(b) Is $C_3 \times C_2$ isomorphic to $C_6$?
(c) Is $C_5 \times C_7$ isomorphic to $C_{35}$?
(d) Is $C_3 \times C_4$ isomorphic to $C_6 \times C_2$?

2. Let $G = D_4$. Let $r$ be the element of order 4 that generates the rotations in $G$ and let $f$ (having order 2) be a flip. Recall that $frf = r^3$. For each subgroup $H$ of $G$ below, do the following: (i) state whether or not $H$ is a normal subgroup of $G$, and (ii) if $H$ is normal, write the factor group $G/H$ as a product of cyclic groups. Explain your reasoning.
(a) $H = \{e, f\}$.
(b) $H = \{e, r, r^2, r^3\}$.
(c) $H = \{e, r^2\}$.

3. (a) Let $G$ be a group. Let $H$ be a subgroup of $G$, and let $N$ be a normal subgroup of $G$. Show that the set $HN = \{hn \mid h \in H, n \in N\}$ is a subgroup of $G$.
(b) Give an example of a group $G$ and two subgroups $H$ and $K$ of $G$ such that the set $HK = \{hk \mid h \in H, k \in K\}$ is not a subgroup of $G$. [Hint: you will need to use two subgroups that are not normal]

4. Let $G$ be a group. Recall that the commutator subgroup $C(G)$ is the subgroup of $G$ generated by all elements of the form $xyx^{-1}y^{-1}$ where $x, y \in G$. Recall that $C(G)$ is a normal subgroup of $G$.
(a) Let $N$ be any normal subgroup of $G$. Show that if the factor group $G/N$ is abelian, then $N$ must contain $C(G)$. [Hint: it suffices to show that $xyx^{-1}y^{-1} \in N$ for all $x, y \in G$]
(b) Use (a) to show that if $G$ is a finite group and $H$ is subgroup of $G$ such that $|G| = 2|H|$, then $H$ must contain $C(G)$.
(c) Use (b) to show that $A_5$ is $C(S_5)$. You may use the fact, proven in your homework, that $A_5$ is simple.

5. Let $G$ be the subgroup of $S_6$ generated by $(135)(46)$, i.e. $G = \langle (135)(46) \rangle$. Let $G$ act on the set $\{1, \ldots, 6\}$ in the usual way, e.g. $(135)(46) * 1 = 3$, etc.
(a) Write down all the orbits in $\{1, \ldots, 6\}$ under the action of $G$.
(b) What is the isotropy group $G_3$ of the element 3?
(b) What is the isotropy group $G_1$ of the element 4?
(b) What is the isotropy group $G_2$ of the element 2?
6. Let $G$ be a group. Let $\varphi_2$ be the map from $G$ to itself given by $\varphi_2(x) = x^2$ for each $x \in G$.
(a) Show that if $G$ is abelian, then $\varphi_2$ is a homomorphism.
(b) Given an example of non-abelian group $G$ for which $\varphi_2$ is not a homomorphism.

**EXTRA CREDIT:** Let $A_4$ act on a set $X$. Suppose that $x \in X$ has an orbit of size 3, that is $|\text{Orb}(x)| = 3$ (or, to use the book’s notation, $|Gx| = 3$). What is the isotropy group $G_x$ of $x$?