1. Let $R$ be a Noetherian domain with the property that every prime ideal is principal. Show that every ideal of $R$ is principal. [Hint: You may want to begin by showing that $R$ is Dedekind]

2. We will say that a ring $R$ is unique factorization domain (UFD) if $R$ is an integral domain and if

- every nonunit $a \in R$ can be written as $\prod_{i=1}^{n} \pi_i^{e_i}$, where $e_i \in \mathbb{Z}^+$ and $R\pi_i$ is a prime ideal in $R$; and
- given two factorizations

$$a = \prod_{i=1}^{n} \pi_i^{e_i} = \prod_{i=1}^{m} \gamma_i^{f_i},$$

where $e_i, f_i \in \mathbb{Z}^+$ and $R\pi_i, R\gamma_i$ are prime ideals in $R$, we must have $m = n$ and a reordering $\sigma$ of $1, \ldots, n$ such that $R\pi_i = R\gamma_{\sigma(i)}$ and $e_i = f_{\sigma(i)}$.

Since a principal ideal domain is a Dedekind domain or a field, it follows from unique factorization for ideals in a Dedekind domain that a principal ideal domain is a UFD. Show the partial converse: any Noetherian UFD of dimension 1 is a principal ideal domain. [You may use Problem #1]

3. Let $F_q$ be a finite field. Let $\overline{F}_q$ be an algebraic closure of $F_q$. Show that $\text{Gal}(\overline{F}_q/F_q)$ is isomorphic to $\hat{\mathbb{Z}}$, where $\hat{\mathbb{Z}}$ is the inverse limit of the groups $A_m$ where $A_m = \mathbb{Z}/m\mathbb{Z}$ where the maps $\mu_{ij}$ are the natural quotient maps from $A_i$ to $A_j$.

AND