The Atiyah-Hirzebruch spectral sequence for $ER(2)^*(\mathbb{C}P^\infty)$.

Sunday 10th September, 2017

One of the computations we did in [KLW17] was to compute the Atiyah-Hirzebruch spectral sequence (AHSS) for $ER(2)^*(\mathbb{C}P^\infty)$ (Section 6). Another was for $ER(2)^*(\mathbb{C}P^n)$ (Section 7).

There were many spectral sequences in the paper and the referee suggested some pictures would help. A pathetic minimal attempt was made after Theorem 6.7.

This little contribution is mainly to develop some familiarity with Hood Chatham's spectral sequence package for LaTeX. Many thanks to him for adding features (fixing bugs?) as I went along.

There are several things to be said about the notation. The theory $ER(2)$ is 48-periodic and we grade it accordingly (from 1-48). The homotopy is displayed nicely in the appendix of the above mentioned paper (never to be mentioned again, but always there). The homotopy is also written down as Fact 2.1 in Section 2, where, unfortunately, the periodicity operator $\hat{v}_2$ is still (unnecessarily) around.

The E_2 term of the AHSS is written down in Theorem 6.1. The AHSS has a 16-periodicity about it, i.e. we only give the x-coordingate up to 16, but if you continue this pattern of 16 indefinitely, you get $\mathbb{C}P^\infty$.

All of the $\mathbb{C}P^{8k+i}$ look alike for different k. One can view the picture we make as the first 16 degrees of $\mathbb{C}P^\infty$ or the last 16 of $\mathbb{C}P^{8k}$. Just chop off the terms on the right to get any desired $\mathbb{C}P^{8k+i}$. What you see is that some differentials near the very top of the space end up going to zero, so certain classes stay around for the finite spaces that don't for $\mathbb{C}P^\infty$.

We give names to all the terms as in the usual AHSS, i.e. $H^*(\mathbb{C}P^\infty; ER(2)^*)$. Some of the key eccentricities to our notation to note are that any generator with an x in it is a $\mathbb{Z}/(2)$. If there is no x, it is a $\mathbb{Z}_{(2)}$.

Sometimes we want 2 times an element to go away. To indicate that the remaining term is a mod 2 class, we write it as (class)/2. (Horrendous notation.)

In an attempt to give a better indication of what is going on, we clean up the AHSS at various stages. Before this computation is started, we already know $ER(2)^*(\mathbb{C}P^\infty)$, which is a great help in getting the differentials, etc. (Theorem 3.1)

$ER(2)^*(\mathbb{C}P^\infty)$ can be written in terms of x-torsion. There are x^1-torsion generators, x^3-torsion generators, and one x^7-torsion generator.

What we do is identify, Theorem 6.2, all of the x^1-torsion generators. We color them red and then make them disappear.

Only after we do that do we show you want d_2 does. It is given by $d_2(u) = x\alpha u^2$ and products (see Theorem 6.1).

Ater d_2 takes elements away, we identify all of the known x^3-torsion generators (in red) and x and x^2 times them in green. All of these stay in the same filtration except for one case, x^6u^3 is a generator and x times it gives an element in higher filtration, xwu^4. (And, of course, there is a corresponding x^6u^7 and xwu^8.) This is from Theorem 6.4.

Then we take away all of the elements associated with x^3-torsion (we know they are not sources or targets in the AHSS).

Next we have that d_4 is determined by $d_4(u^2) = x^3u^4$ and products. Theorem 6.3.

After d_4 kills off its elements, we identify the only x^7-torsion generator (in red) and the x^{1-6} times it in green.

After that, we remove them and compute d_6, which is determined by $d_6(x^4u^3) = xwu^6$ (and products).

Our reward now is that there is nothing left.

Hope this helps.

	2	4	6	8	10	12	14	16
49								
48	$\alpha^{3k}u^1$	$\alpha^{3k}u^2$	$\alpha^{3k}u^3$	$\alpha^{3k}u^4$	$\alpha^{3k}u^5$	$\alpha^{3k}u^6$	$\alpha^{3k}u^7$	$\alpha^{3k}u^8$
47	$x\alpha^{3k+1}u^1$	$x\alpha^{3k+1}u^2$	$x\alpha^{3k+1}u^3$	$x\alpha^{3k+1}u^4$	$x\alpha^{3k+1}u^5$	$x\alpha^{3k+1}u^6$	$x\alpha^{3k+1}u^7$	$x\alpha^{3k+1}u^8$
46	$x^2\alpha^{3k+2}u^1$	$x^2\alpha^{3k+2}u^2$	$x^2\alpha^{3k+2}u^3$	$x^2\alpha^{3k+2}u^4$	$x^2\alpha^{3k+2}u^5$	$x^2\alpha^{3k+2}u^6$	$x^2\alpha^{3k+2}u^7$	$x^2\alpha^{3k+2}u^8$
45	x^3u^1	x^3u^2	x^3u^3	x^3u^4	x^3u^5	x^3u^6	x^3u^7	x^3u^8
44	$\alpha_3\alpha^{3k+2}u^1$	$\alpha_3\alpha^{3k+2}u^2$	$\alpha_3\alpha^{3k+2}u^3$	$\alpha_3\alpha^{3k+2}u^4$	$\alpha_3\alpha^{3k+2}u^5$	$\alpha_3\alpha^{3k+2}u^6$	$\alpha_3\alpha^{3k+2}u^7$	$\alpha_3\alpha^{3k+2}u^8$
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42	x^6u^1	x^6u^2	x^6u^3	x^6u^4	x^6u^5	x^6u^6	x^6u^7	x^6u^8
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40	$w\alpha^{3k}u^1$	$w\alpha^{3k}u^2$	$w\alpha^{3k}u^3$	$w\alpha^{3k}u^4$	$w\alpha^{3k}u^5$	$w\alpha^{3k}u^6$	$w\alpha^{3k}u^7$	$w\alpha^{3k}u^8$
39	$xw\alpha^{3k+1}u^1$	$xw\alpha^{3k+1}u^2$	$xw\alpha^{3k+1}u^3$	$xw\alpha^{3k+1}u^4$	$xw\alpha^{3k+1}u^5$	$xw\alpha^{3k+1}u^6$	$xw\alpha^{3k+1}u^7$	$xw\alpha^{3k+1}u^8$
38	$x^2w\alpha^{3k+2}u^1$	$x^2w\alpha^{3k+2}u^2$	$x^2w\alpha^{3k+2}u^3$	$x^2w\alpha^{3k+2}u^4$	$x^2w\alpha^{3k+2}u^5$	$x^2w\alpha^{3k+2}u^6$	$x^2w\alpha^{3k+2}u^7$	$x^2w\alpha^{3k+2}u^8$
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31	$x\alpha^{3k}u^1$	$x\alpha^{3k}u^2$	$x\alpha^{3k}u^3$	$x\alpha^{3k}u^4$	$x\alpha^{3k}u^5$	$x\alpha^{3k}u^6$	$x\alpha^{3k}u^7$	$x\alpha^{3k}u^8$
30	$x^2\alpha^{3k+1}u^1$	$x^2\alpha^{3k+1}u^2$	$x^2\alpha^{3k+1}u^3$	$x^2\alpha^{3k+1}u^4$	$x^2\alpha^{3k+1}u^5$	$x^2\alpha^{3k+1}u^6$	$x^2\alpha^{3k+1}u^7$	$x^2\alpha^{3k+1}u^8$
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28	$\alpha_3\alpha^{3k+1}u^1$ x^4u^1	$\alpha_3\alpha^{3k+1}u^2$ x^4u^2	$\alpha_3\alpha^{3k+1}u^3$ x^4u^3	$\alpha_3\alpha^{3k+1}u^4$ x^4u^4	$\alpha_3\alpha^{3k+1}u^5$ x^4u^5	$\alpha_3\alpha^{3k+1}u^6$ x^4u^6	$\alpha_3\alpha^{3k+1}u^7$ x^4u^7	$\alpha_3\alpha^{3k+1}u^8$ x^4u^8
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23	$xw\alpha^{3k}u^1$	$xw\alpha^{3k}u^2$	$xw\alpha^{3k}u^3$	$xw\alpha^{3k}u^4$	$xw\alpha^{3k}u^5$	$xw\alpha^{3k}u^6$	$xw\alpha^{3k}u^7$	$xw\alpha^{3k}u^8$
22	$x^2w\alpha^{3k+1}u^1$	$x^2w\alpha^{3k+1}u^2$	$x^2w\alpha^{3k+1}u^3$	$x^2w\alpha^{3k+1}u^4$	$x^2w\alpha^{3k+1}u^5$	$x^2w\alpha^{3k+1}u^6$	$x^2w\alpha^{3k+1}u^7$	$x^2w\alpha^{3k+1}u^8$
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15	$x\alpha^{3k+2}u^1$	$x\alpha^{3k+2}u^2$	$x\alpha^{3k+2}u^3$	$x\alpha^{3k+2}u^4$	$x\alpha^{3k+2}u^5$	$x\alpha^{3k+2}u^6$	$x\alpha^{3k+2}u^7$	$x\alpha^{3k+2}u^8$
14	$x^2\alpha^{3k}u^1$	$x^2\alpha^{3k}u^2$	$x^2\alpha^{3k}u^3$	$x^2\alpha^{3k}u^4$	$x^2\alpha^{3k}u^5$	$x^2\alpha^{3k}u^6$	$x^2\alpha^{3k}u^7$	$x^2\alpha^{3k}u^8$
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11	x^5u^1	x^5u^2	x^5u^3	x^5u^4	x^5u^5	x^5u^6	x^5u^7	x^5u^8
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7	$xw\alpha^{3k+2}u^1$	$xw\alpha^{3k+2}u^2$	$xw\alpha^{3k+2}u^3$	$xw\alpha^{3k+2}u^4$	$xw\alpha^{3k+2}u^5$	$xw\alpha^{3k+2}u^6$	$xw\alpha^{3k+2}u^7$	$xw\alpha^{3k+2}u^8$
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40	$2w\alpha^{3k}u^1$ $w\alpha^{3k}u^1/2$	$w\alpha^{3k}u^2$	$2w\alpha^{3k}u^3$ $w\alpha^{3k}u^3/2$	$w\alpha^{3k}u^4$	$2w\alpha^{3k}u^5$ $w\alpha^{3k}u^5/2$	$w\alpha^{3k}u^6$	$2w\alpha^{3k}u^7$ $w\alpha^{3k}u^7/2$	$w\alpha^{3k}u^8$
39	$xw\alpha^{3k+1}u^1$	$xw\alpha^{3k+1}u^2$	$xw\alpha^{3k+1}u^3$	$xw\alpha^{3k+1}u^4$	$xw\alpha^{3k+1}u^5$	$xw\alpha^{3k+1}u^6$	$xw\alpha^{3k+1}u^7$	$xw\alpha^{3k+1}u^8$
38	$x^2w\alpha^{3k+2}u^1$	$x^2w\alpha^{3k+2}u^2$	$x^2w\alpha^{3k+2}u^3$	$x^2w\alpha^{3k+2}u^4$	$x^2w\alpha^{3k+2}u^5$	$x^2w\alpha^{3k+2}u^6$	$x^2w\alpha^{3k+2}u^7$	$x^2w\alpha^{3k+2}u^8$
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32	$2\alpha^{3k+2}u^1$ $\alpha^{3k+2}u^1/2$	$\alpha^{3k+2}u^2$	$2\alpha^{3k+2}u^3$ $\alpha^{3k+2}u^3/2$	$\alpha^{3k+2}u^4$	$2\alpha^{3k+2}u^5$ $\alpha^{3k+2}u^5/2$	$\alpha^{3k+2}u^6$	$2\alpha^{3k+2}u^7$ $\alpha^{3k+2}u^7/2$	$\alpha^{3k+2}u^8$
31	$x\alpha^{3k}u^1$	$x\alpha^{3k}u^2$	$x\alpha^{3k}u^3$	$x\alpha^{3k}u^4$	$x\alpha^{3k}u^5$	$x\alpha^{3k}u^6$	$x\alpha^{3k}u^7$	$x\alpha^{3k}u^8$
30	$x^2\alpha^{3k+1}u^1$	$x^2\alpha^{3k+1}u^2$	$x^2\alpha^{3k+1}u^3$	$x^2\alpha^{3k+1}u^4$	$x^2\alpha^{3k+1}u^5$	$x^2\alpha^{3k+1}u^6$	$x^2\alpha^{3k+1}u^7$	$x^2\alpha^{3k+1}u^8$
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28	$\alpha_3\alpha^{3k+1}u^1$ x^4u^1	$\alpha_3\alpha^{3k+1}u^2$ x^4u^2	$\alpha_3\alpha^{3k+1}u^3$ x^4u^3	$\alpha_3\alpha^{3k+1}u^4$ x^4u^4	$\alpha_3\alpha^{3k+1}u^5$ x^4u^5	$\alpha_3\alpha^{3k+1}u^6$ x^4u^6	$\alpha_3\alpha^{3k+1}u^7$ x^4u^7	$\alpha_3\alpha^{3k+1}u^8$ x^4u^8
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23	$xw\alpha^{3k}u^1$	$xw\alpha^{3k}u^2$	$xw\alpha^{3k}u^3$	$xw\alpha^{3k}u^4$	$xw\alpha^{3k}u^5$	$xw\alpha^{3k}u^6$	$xw\alpha^{3k}u^7$	$xw\alpha^{3k}u^8$
22	$x^2w\alpha^{3k+1}u^1$	$x^2w\alpha^{3k+1}u^2$	$x^2w\alpha^{3k+1}u^3$	$x^2w\alpha^{3k+1}u^4$	$x^2w\alpha^{3k+1}u^5$	$x^2w\alpha^{3k+1}u^6$	$x^2w\alpha^{3k+1}u^7$	$x^2w\alpha^{3k+1}u^8$
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20	$\alpha_1\alpha^{3k+2}u^1$	$\alpha_1\alpha^{3k+2}u^2$	$\alpha_1\alpha^{3k+2}u^3$	$\alpha_1\alpha^{3k+2}u^4$	$\alpha_1\alpha^{3k+2}u^5$	$\alpha_1\alpha^{3k+2}u^6$	$\alpha_1\alpha^{3k+2}u^7$	$\alpha_1\alpha^{3k+2}u^8$
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16	$2\alpha^{3k+1}u^1$ $\alpha^{3k+1}u^1/2$	$\alpha^{3k+1}u^2$	$2\alpha^{3k+1}u^3$ $\alpha^{3k+1}u^3/2$	$\alpha^{3k+1}u^4$	$2\alpha^{3k+1}u^5$ $\alpha^{3k+1}u^5/2$	$\alpha^{3k+1}u^6$	$2\alpha^{3k+1}u^7$ $\alpha^{3k+1}u^7/2$	$\alpha^{3k+1}u^8$
15	$x\alpha^{3k+2}u^1$	$x\alpha^{3k+2}u^2$	$x\alpha^{3k+2}u^3$	$x\alpha^{3k+2}u^4$	$x\alpha^{3k+2}u^5$	$x\alpha^{3k+2}u^6$	$x\alpha^{3k+2}u^7$	$x\alpha^{3k+2}u^8$
14	$x^2\alpha^{3k+3}u^1$ x^2u^1	$x^2\alpha^{3k}u^2$	$x^2\alpha^{3k+3}u^3$ x^2u^3	$x^2\alpha^{3k}u^4$	$x^2\alpha^{3k+3}u^5$ x^2u^5	$x^2\alpha^{3k}u^6$	$x^2\alpha^{3k+3}u^7$ x^2u^7	$x^2\alpha^{3k}u^8$
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12	$\alpha_3\alpha^{3k}u^1$	$\alpha_3\alpha^{3k}u^2$	$\alpha_3\alpha^{3k}u^3$	$\alpha_3\alpha^{3k}u^4$	$\alpha_3\alpha^{3k}u^5$	$\alpha_3\alpha^{3k}u^6$	$\alpha_3\alpha^{3k}u^7$	$\alpha_3\alpha^{3k}u^8$
11	x^5u^1	x^5u^2	x^5u^3	x^5u^4	x^5u^5	x^5u^6	x^5u^7	x^5u^8
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8	$2w\alpha^{3k+1}u^1$ $w\alpha^{3k+1}u^1/2$	$w\alpha^{3k+1}u^2$	$2w\alpha^{3k+1}u^3$ $w\alpha^{3k+1}u^3/2$	$w\alpha^{3k+1}u^4$	$2w\alpha^{3k+1}u^5$ $w\alpha^{3k+1}u^5/2$	$w\alpha^{3k+1}u^6$	$2w\alpha^{3k+1}u^7$ $w\alpha^{3k+1}u^7/2$	$w\alpha^{3k+1}u^8$
7	$xw\alpha^{3k+2}u^1$	$xw\alpha^{3k+2}u^2$	$xw\alpha^{3k+2}u^3$	$xw\alpha^{3k+2}u^4$	$xw\alpha^{3k+2}u^5$	$xw\alpha^{3k+2}u^6$	$xw\alpha^{3k+2}u^7$	$xw\alpha^{3k+2}u^8$
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5								
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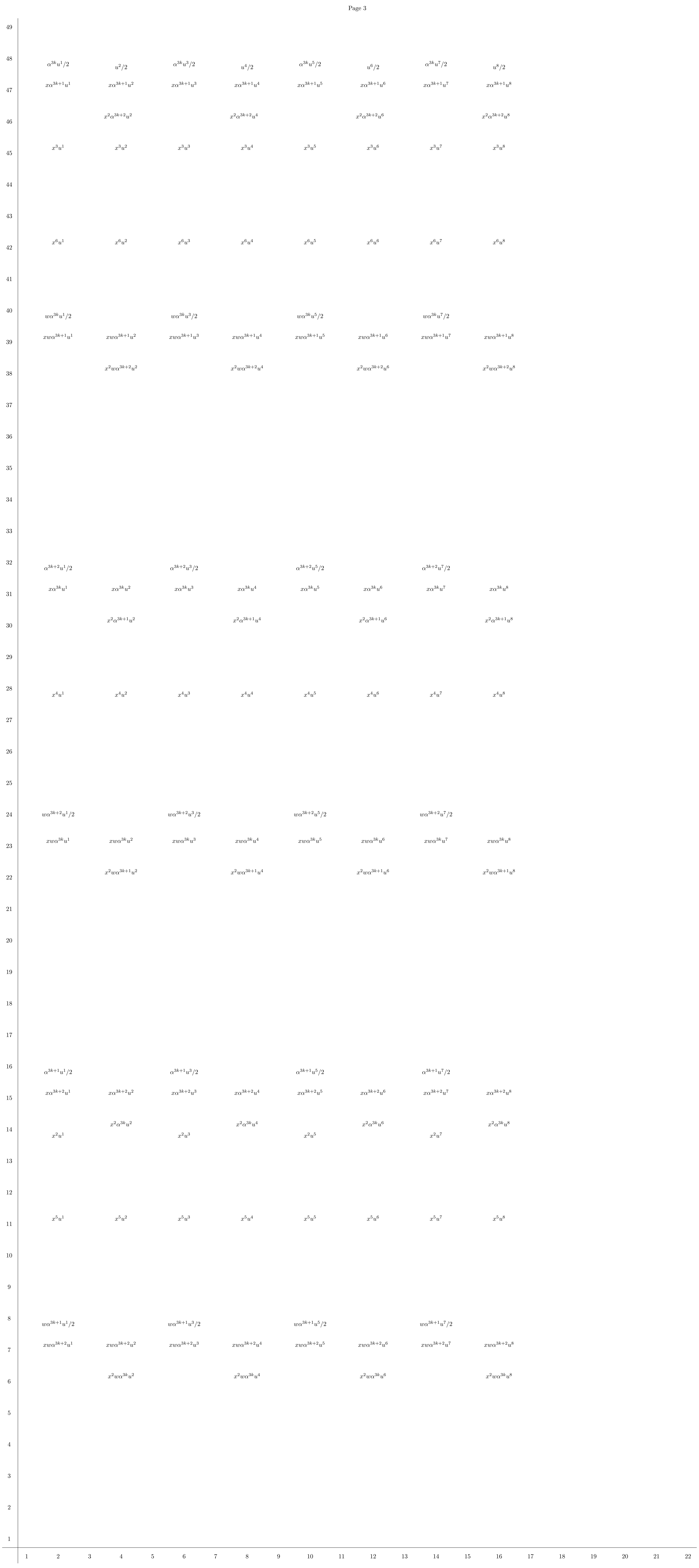
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$x^2\alpha^{3k+2}u^2$ $x^2\alpha^{3k+2}u^4$ $x^2\alpha^{3k+2}u^6$ $x^2\alpha^{3k+2}u^8$
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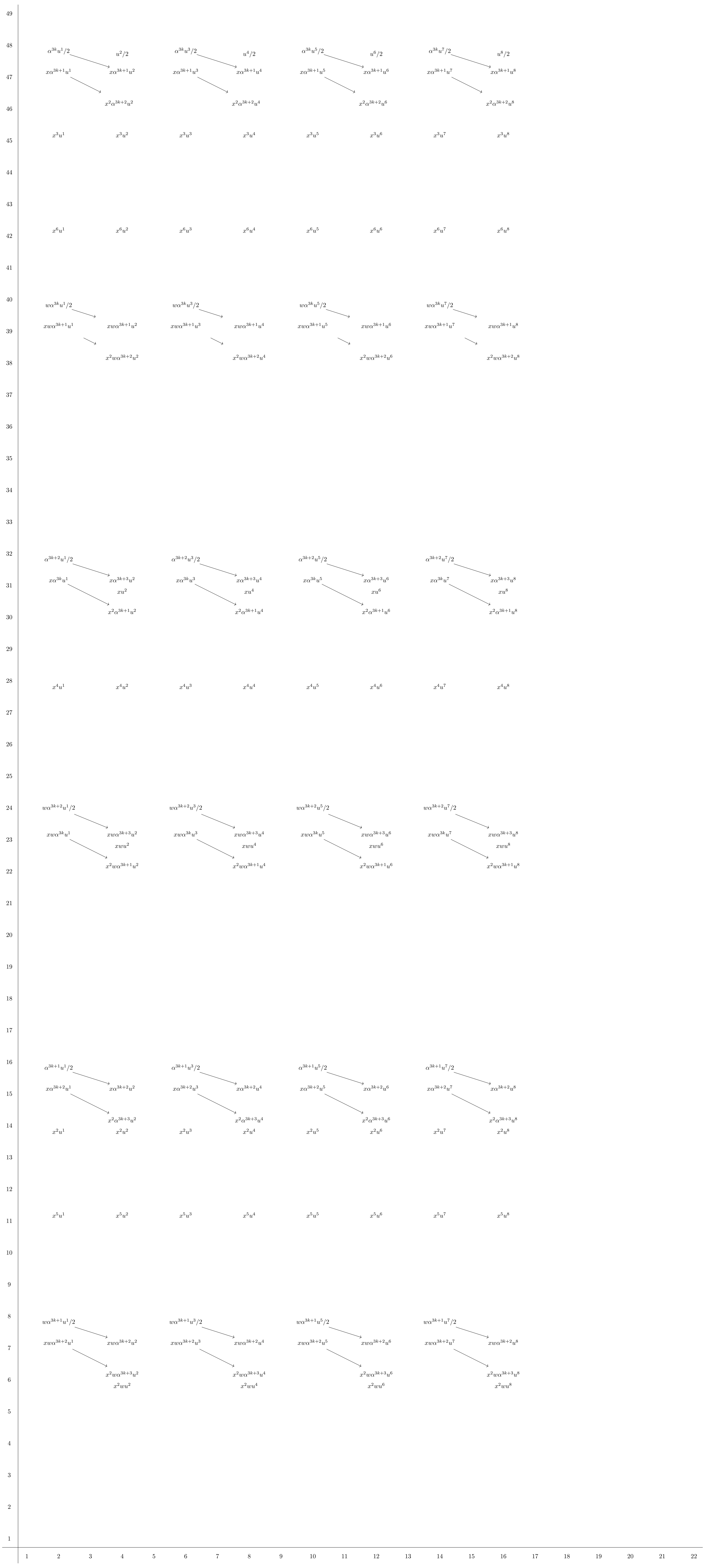
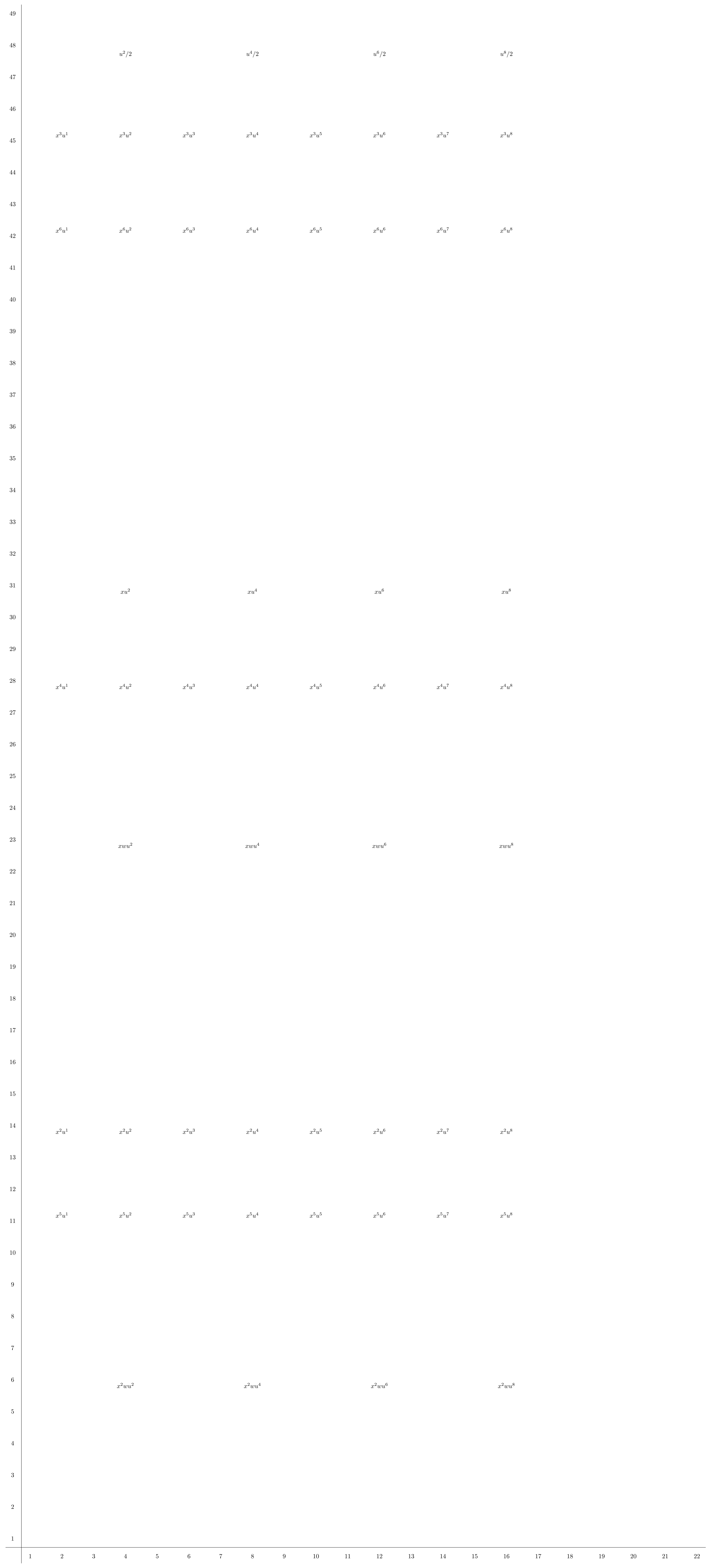
Page 4

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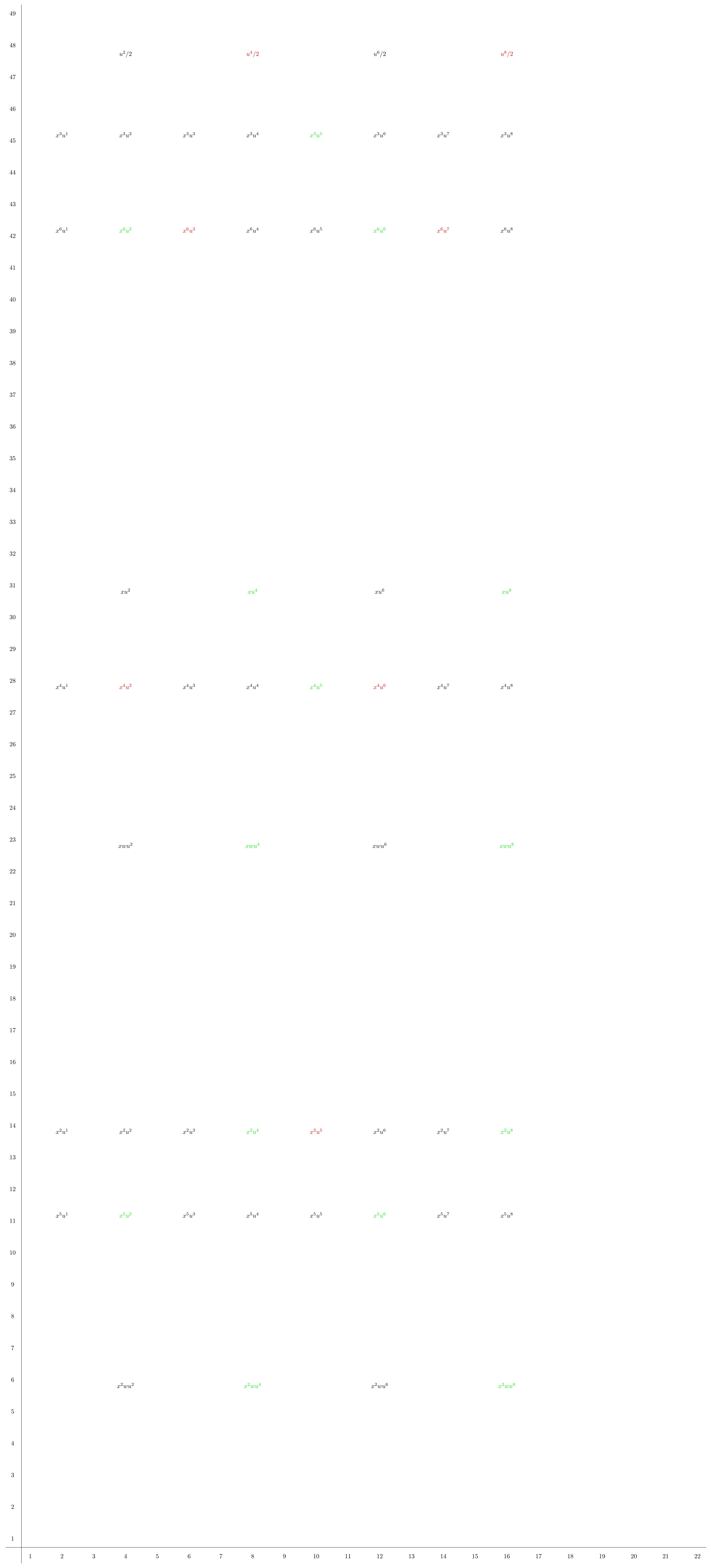
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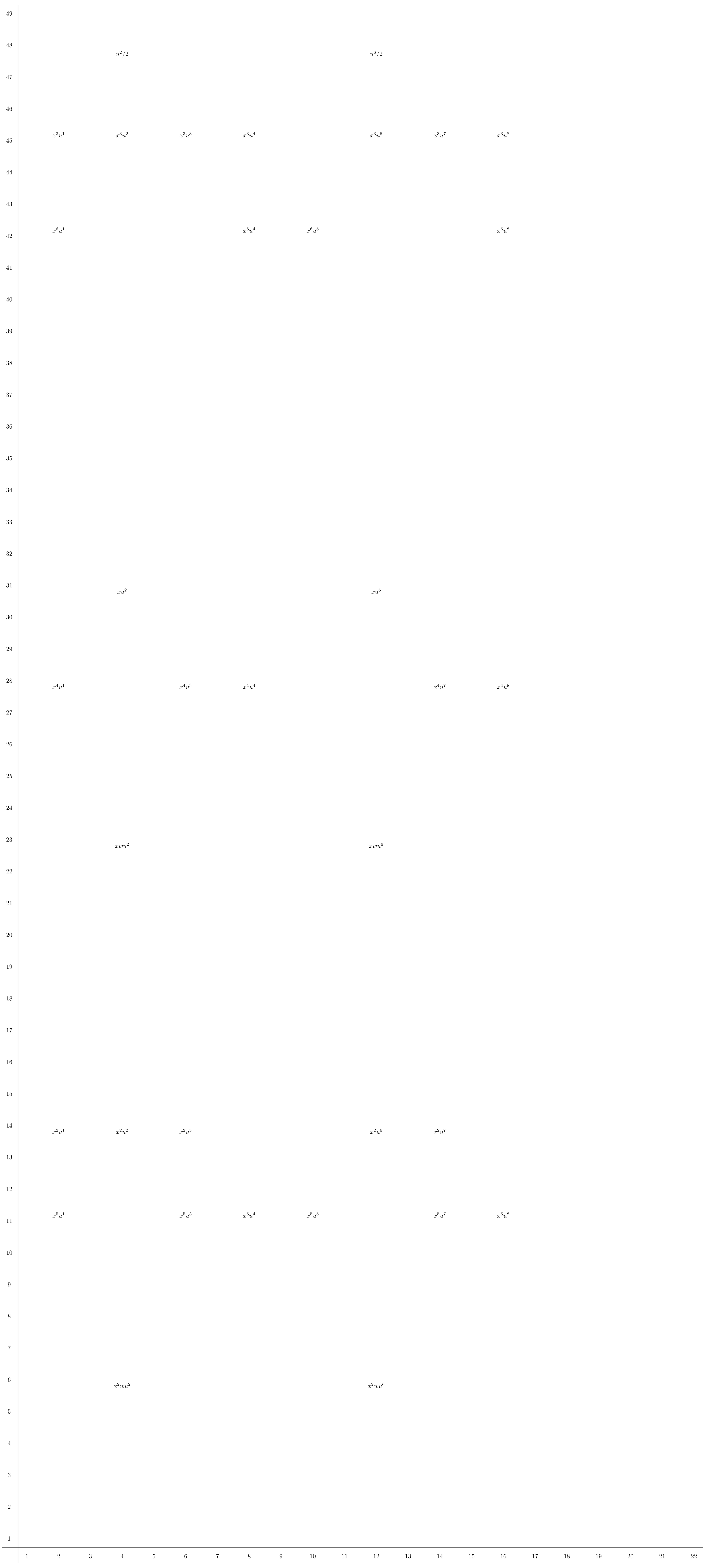
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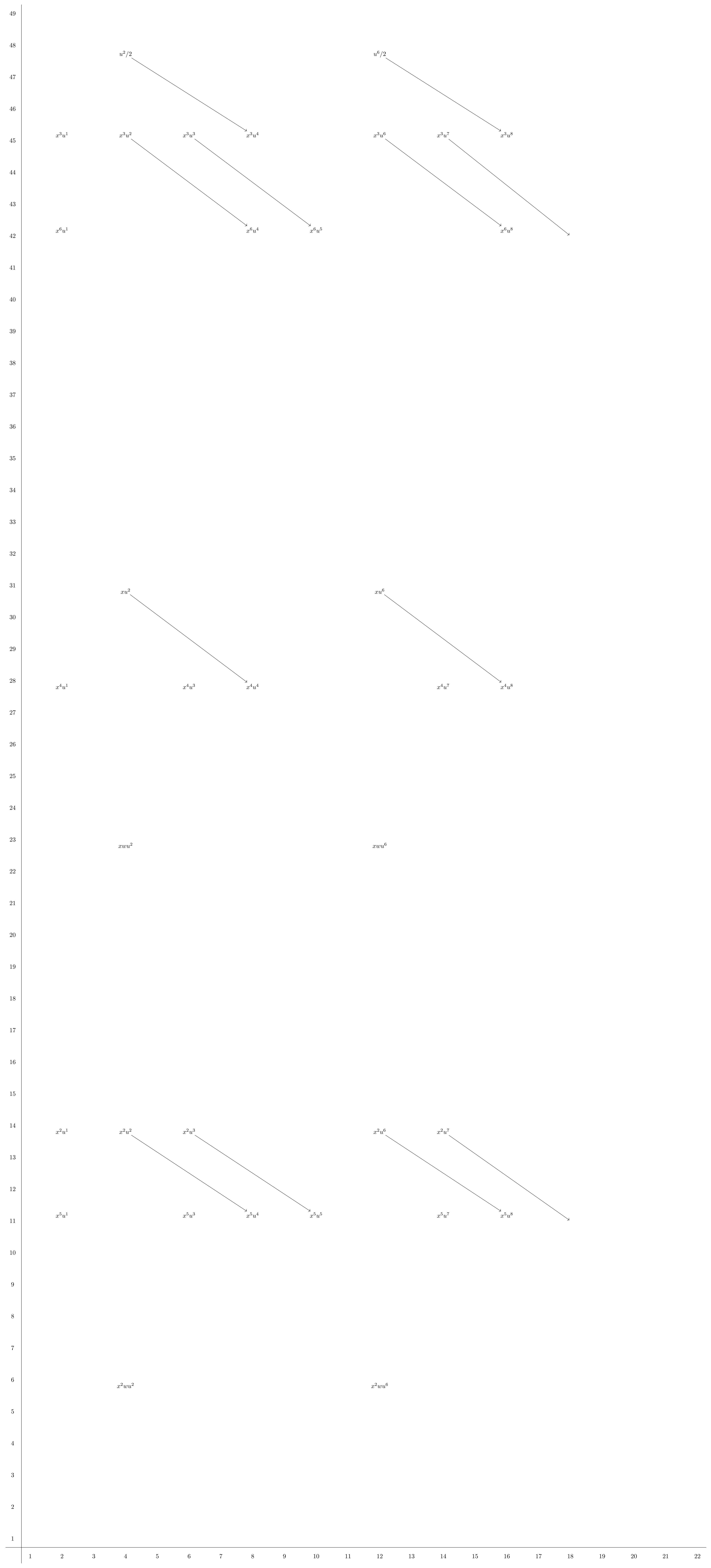
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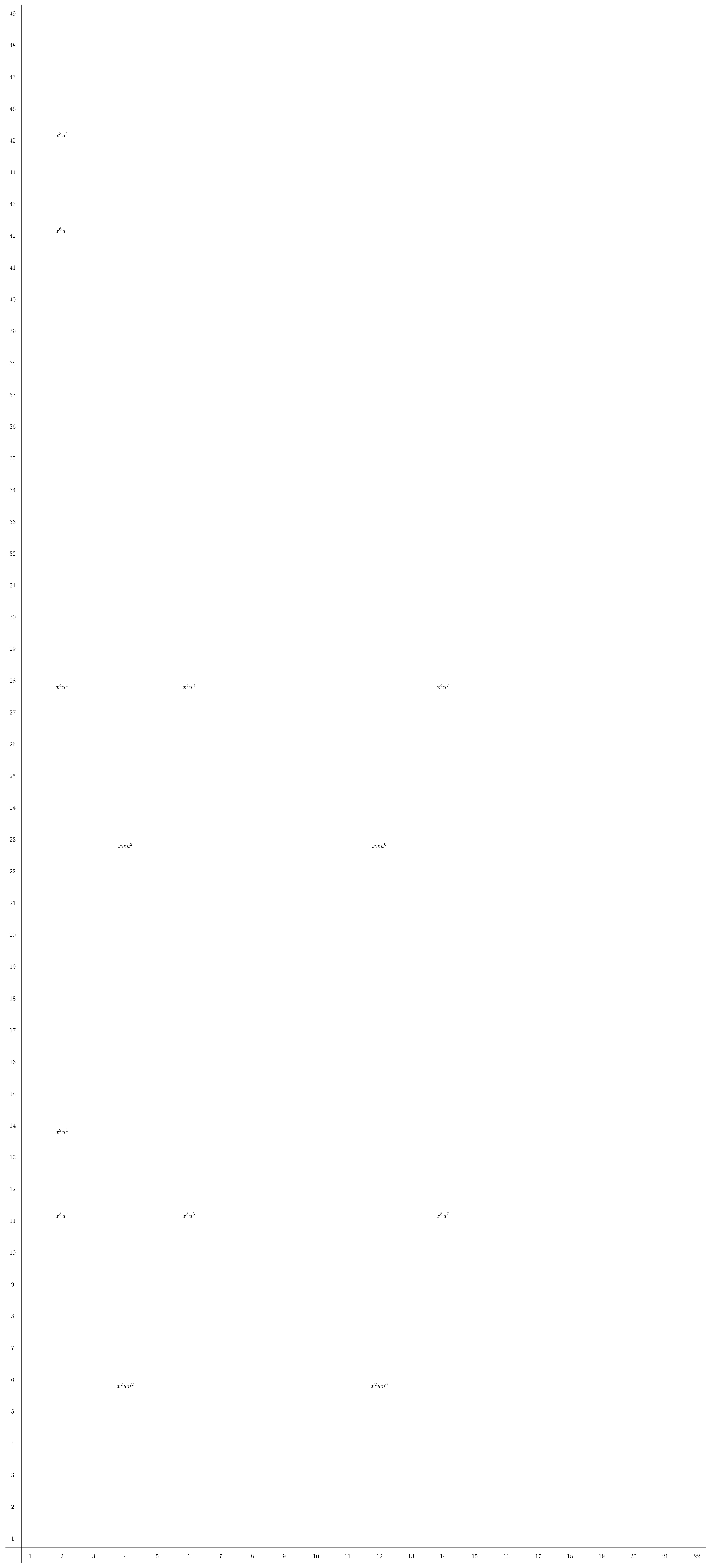
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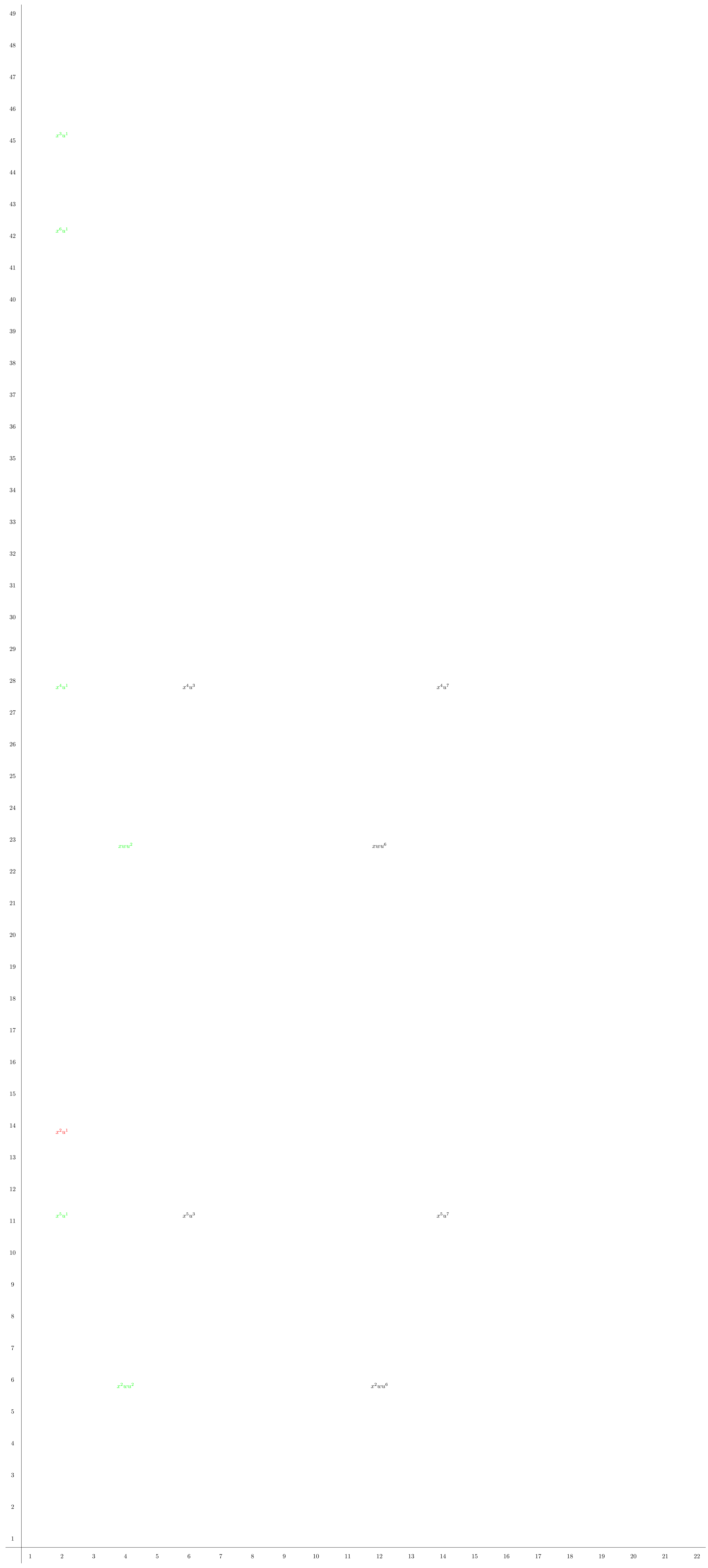
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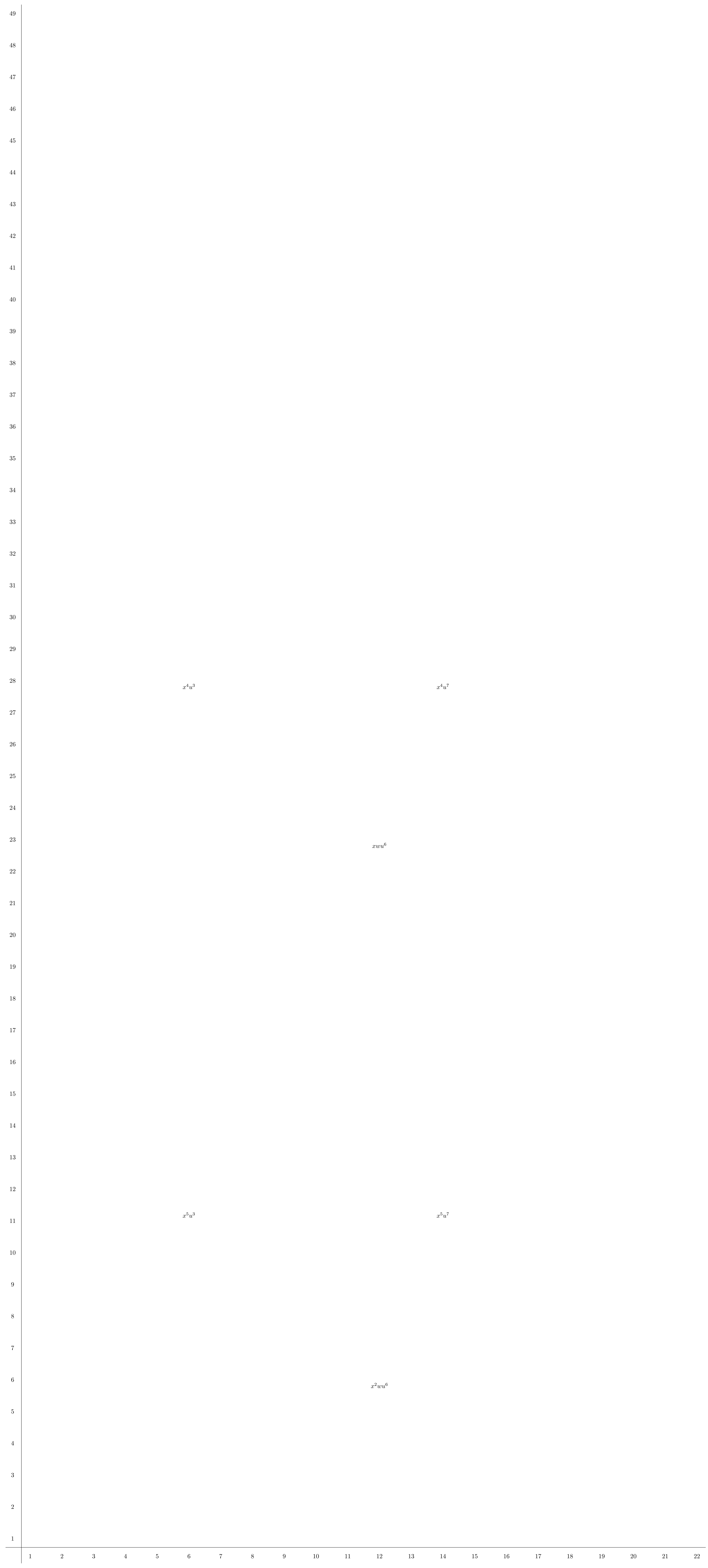
Page 11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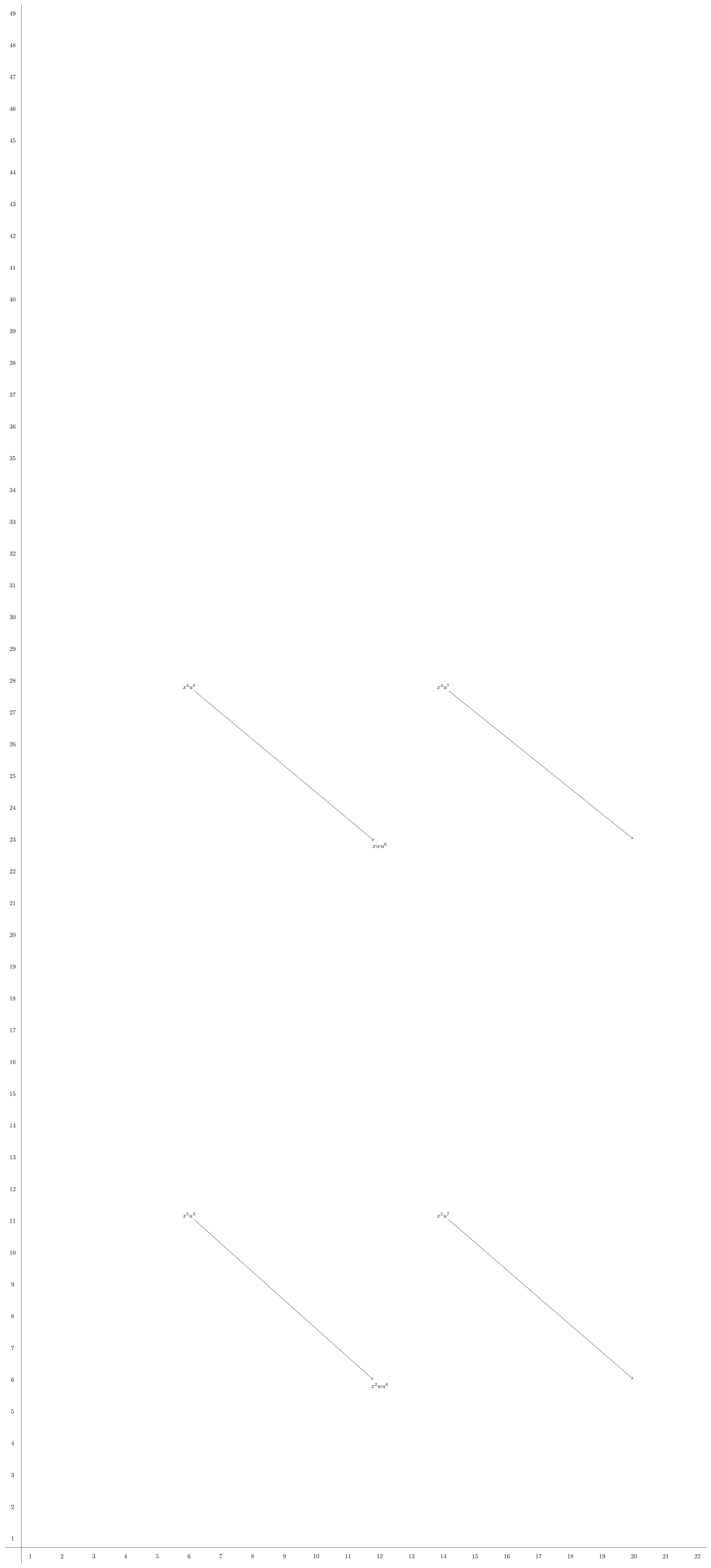
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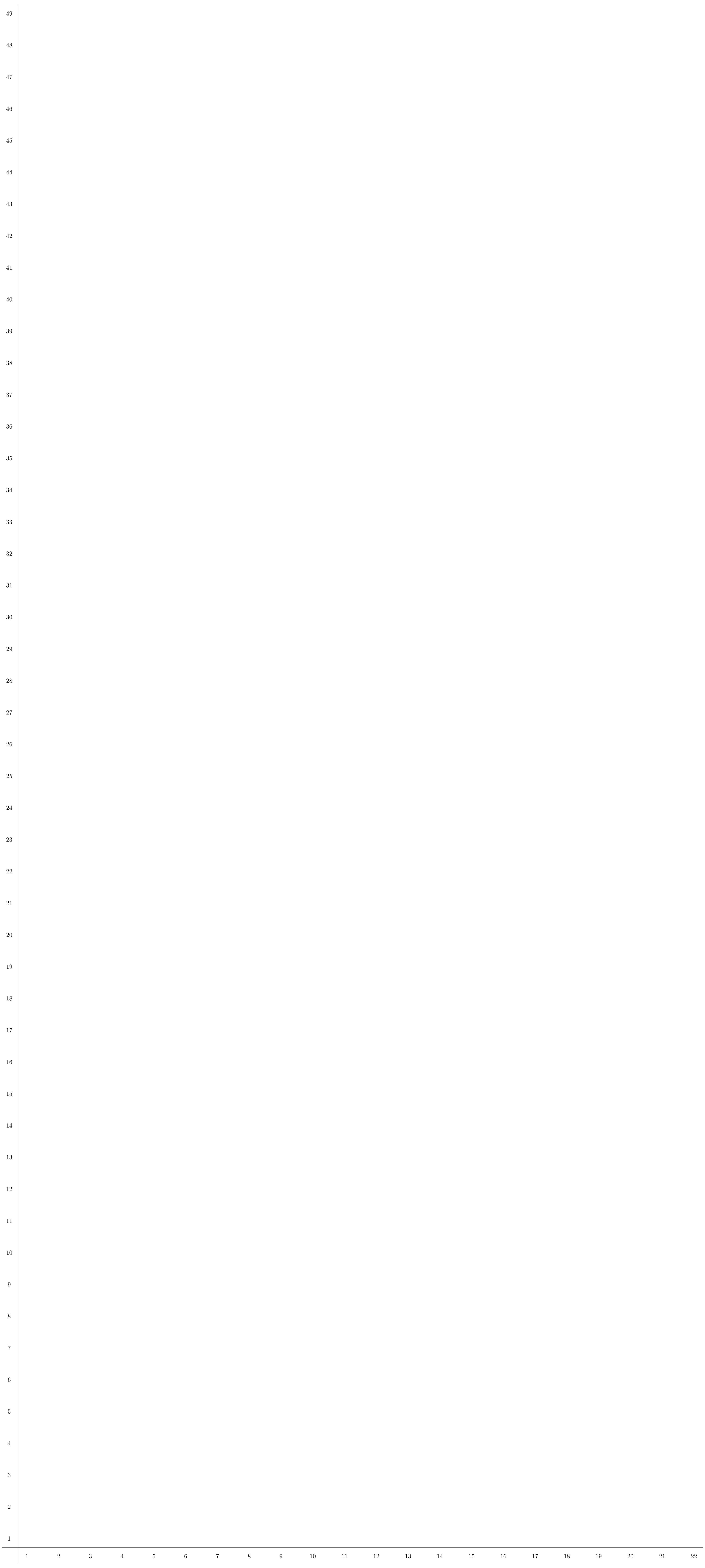
Page 13



References

[KLW17] N. Kitchloo, V. Lorman, and W.S. Wilson. The $ER(2)$-cohomology of $BZ/(2^q)$ and CP^n. *Canadian Journal of Mathematics*, 2017. To appear.