Math 162: Calculus IIA
Final Exam
December 15, 2015

NAME (please print legibly): ____________________________________________
Your University ID Number: __________________________________________
Your University email _________________________________________________
Indicate your instructor with a check in the box:

<table>
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<tr>
<th>Instructor</th>
<th>Class Time</th>
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<tbody>
<tr>
<td>JJ Lee</td>
<td>MWF 9:00 - 9:50 AM</td>
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<tr>
<td>Doug Ravenel</td>
<td>MWF 10:25 - 11:15 AM</td>
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<tr>
<td>Timur Akhunov</td>
<td>MW 12:30 - 1:45 PM</td>
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<tr>
<td>Eyal Neuman</td>
<td>MW 4:50-6:05 PM</td>
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Pledge of Honesty
I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.
Signature: ____________________________________________________________

• The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

• Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. Put your answers in the space provided at the bottom of each page or half page.

• You are responsible for checking that this exam has all 20 pages.

• Part A covers the same material as the two midterms, and Part B covers additional material. Letter grades will be computed for the two parts separately. Part B will count for 20% of your course grade. It has the same weight as a midterm exam grade. Part A will count for at least 10% of your course grade. If your grade on part A is better than your lowest midterm exam grade, then it will replace that midterm exam grade and count for 30% of your course grade.

• Have a nice winter break!
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Part A
1. (15 points) Evaluate the integral

\[ \int \frac{1}{x^2 \sqrt{x^2 + 16}} \, dx. \]
2. (20 points)

(a) Compute the volume of a region bounded by the curves $y = x^3 + 1, y = 1$ and $x = 1$ and rotated around the $x$-axis.
(b) Set up the integral for the volume of the region bounded by \( y = x^4 \), \( y = 0 \) and \( x = 2 \) and rotated around the \( x \)-axis. Use the washer method. Do not evaluate the integral.

**ANSWER:**
3. (10 points)

Evaluate the integral

$$\int (\ln x)^2 \, dx.$$

\textbf{ANSWER:}
4. (20 points)

(a) Find the partial fraction decomposition of

\[
\frac{3x - 2}{x^2 - x}.
\]

ANSWER:
(b) Write out the form of the partial fraction decomposition of the function
\[
\frac{2 + x^3}{x^5 + 2x^3 + x} = \quad \text{[Blank]}
\]

Do not determine the numerical values of the coefficients.

ANSWER:
(c) Let

\[ f(x) = \frac{1}{x} + \frac{4x + 5}{x^2 + 1}. \]

Evaluate

\[ \int f(x) \, dx. \]
5. (15 points)

Use the polar area formula to find the area of one leaf of the three leafed rose, the polar curve defined by \( r = \sin 3\theta \), that is the area for \( 0 \leq \theta \leq \pi/3 \).

\[ \]
6. (20 points)

Find the arc length of the astroid, the parametric curve defined by $x = \cos^3 t$ and $y = \sin^3 t$ for $0 \leq t \leq 2\pi$.

ANSWER:
Part B
7. (20 points)

(a) Find a power series representation centered at 1 as well as the radius and interval of convergence for the function

\[ f(x) = \frac{2(x - 1)}{1 + 2(x - 1)^2}. \]
(b) Write the following integral as a power series in $x$. What is the radius of convergence of this power series?

\[
\int \frac{2(x - 1)}{1 + 2(x - 1)^2} dx
\]
8. (20 points)

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n - 2\sqrt{n} + 2} \]
9. (20 points)

Find the radius of convergence and interval of convergence of the series

\[ \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{4^n(n+1)}. \]
10. (20 points)

(a) Find the Taylor series centered at 0 of the function

\[ g(x) = \tan^{-1}(x^2) - x^2, \]

as well as the radius of convergence.
(b) Write the derivative of \( g(x) \) as a power series and use it to calculate

\[
\frac{dg(x)}{dx} \bigg|_{x=0}
\]
11. (20 points)

(a) Determine whether the series
\[ \sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^n} \]
is absolutely convergent, conditionally convergent, or divergent.

Hint: You may use the fact that \( \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e. \)
(b) Estimate the sum of the series with an accuracy of $\frac{1}{10}$.

ANSWER: