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68

GRADUATE RECORD EXAMINATIONS®

MATHEMATICS TEST

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MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratch work may be done in this examination book.

Note: In this examination:

(1) All logarithms with an unspecified base are natural logarithms, that is, with base e.
(2) The set of all real numbers x such that $a \le x \le b$ is denoted by $[a, b]$.
(3) The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

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1. In the xy-plane, the curve with parametric equations $x = \cos t$ and $y = \sin t$, $0 \le t \le \pi$, has length

(A) 3 (B) π (C) 3π (D) $\frac{3}{2}$ (E) $\frac{\pi}{2}$

---

2. Which of the following is an equation of the line tangent to the graph of $y = x + e^x$ at $x = 0$?

(A) $y = x$

(B) $y = x + 1$

(C) $y = x + 2$

(D) $y = 2x$

(E) $y = 2x + 1$

---

3. If V and W are 2-dimensional subspaces of $\mathbb{R}^4$, what are the possible dimensions of the subspace $V \cap W$?

(A) 1 only (B) 2 only (C) 0 and 1 only (D) 0, 1, and 2 only (E) 0, 1, 2, 3, and 4

---

4. Let k be the number of real solutions of the equation $e^x + x - 2 = 0$ in the interval $[0, 1]$, and let n be the number of real solutions that are not in $[0, 1]$. Which of the following is true?

(A) $k = 0$ and $n = 1$ (B) $k = 1$ and $n = 0$ (C) $k = n = 1$ (D) $k > 1$ (E) $n > 1$

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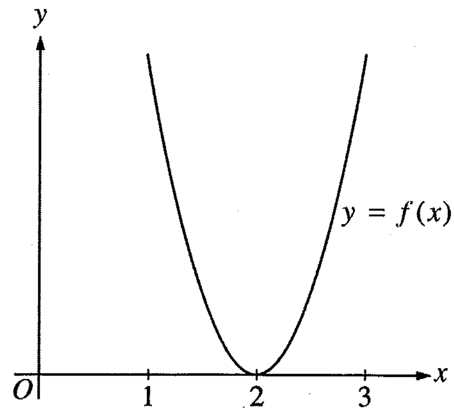

5. Suppose b is a real number and $f(x) = 3x^2 + bx + 12$ defines a function on the real line, part of which is graphed above. Then $f(5) =$

(A) 15 (B) 27 (C) 67 (D) 72 (E) 87

---

6. Which of the following circles has the greatest number of points of intersection with the parabola $x^2 = y + 4$?

(A) $x^2 + y^2 = 1$

(B) $x^2 + y^2 = 2$

(C) $x^2 + y^2 = 9$

(D) $x^2 + y^2 = 16$

(E) $x^2 + y^2 = 25$

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7. $\int_{-3}^{3} |x+1|\, dx =$

(A) 0 (B) 5 (C) 10 (D) 15 (E) 20

---

8. What is the greatest possible area of a triangular region with one vertex at the center of a circle of radius 1 and the other two vertices on the circle?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) π (E) $\frac{1+\sqrt{2}}{4}$

$$J = \int_0^1 \sqrt{1-x^4}\, dx$$
$$K = \int_0^1 \sqrt{1+x^4}\, dx$$
$$L = \int_0^1 \sqrt{1-x^8}\, dx$$

9. Which of the following is true for the definite integrals shown above?

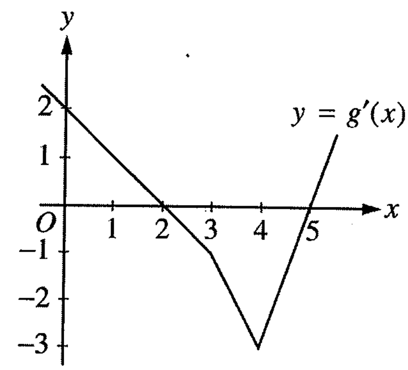
(A) $J < L < 1 < K$

(B) $J < L < K < 1$

(C) $L < J < 1 < K$

(D) $L < J < K < 1$

(E) $L < 1 < J < K$



10. Let g be a function whose derivative g' is continuous and has the graph shown above. Which of the followi[ng] values of g is largest?

(A) $g(1)$ (B) $g(2)$ (C) $g(3)$ (D) $g(4)$ (E) $g(5)$

---

11. Of the following, which is the best approximation of $\sqrt{1.5}\,(266)^{3/2}$?

(A) 1,000 (B) 2,700 (C) 3,200 (D) 4,100 (E) 5,300

---

12. Let A be a 2×2 matrix for which there is a constant k such that the sum of the entries in each row and each column is k. Which of the following must be an eigenvector of A ?

I. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

II. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

III. $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

GO ON TO THE NEXT PAGE

13. A total of x feet of fencing is to form three sides of a level rectangular yard. What is the maximum possible are of the yard, in terms of x ?

(A) $\frac{x^2}{9}$ (B) $\frac{x^2}{8}$ (C) $\frac{x^2}{4}$ (D) x^2 (E) $2x^2$

---

14. What is the units digit in the standard decimal expansion of the number 7^{25} ?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

---

15. Let f be a continuous real-valued function defined on the closed interval $[-2, 3]$. Which of the following is NOT necessarily true?

(A) f is bounded.

(B) $\int_{-2}^{3} f(t)\, dt$ exists.

(C) For each c between $f(-2)$ and $f(3)$, there is an $x \in [-2, 3]$ such that $f(x) = c$.

(D) There is an M in $f([-2, 3])$ such that $\int_{-2}^{3} f(t)\, dt = 5M$.

(E) $\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ exists.

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16. What is the volume of the solid formed by revolving about the x-axis the region in the first quadrant of the xy-plane bounded by the coordinate axes and the graph of the equation $y = \frac{1}{\sqrt{1+x^2}}$?

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi^2}{4}$ (D) $\frac{\pi^2}{2}$ (E) ∞

---

17. How many real roots does the polynomial $2x^5 + 8x - 7$ have?

(A) None (B) One (C) Two (D) Three (E) Five

---

18. Let V be the real vector space of all real 2×3 matrices, and let W be the real vector space of all real 4×1 column vectors. If T is a linear transformation from V onto W, what is the dimension of the subspace $\{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}\}$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

---

19. Let f and g be twice-differentiable real-valued functions defined on $\mathbb{R}$. If $f'(x) > g'(x)$ for all $x > 0$, which of the following inequalities must be true for all $x > 0$?

(A) $f(x) > g(x)$

(B) $f''(x) > g''(x)$

(C) $f(x) - f(0) > g(x) - g(0)$

(D) $f'(x) - f'(0) > g'(x) - g'(0)$

(E) $f''(x) - f''(0) > g''(x) - g''(0)$

---

20. Let f be the function defined on the real line by

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is rational} \\ \frac{x}{3} & \text{if } x \text{ is irrational.} \end{cases}$$

If D is the set of points of discontinuity of f, then D is the

(A) empty set

(B) set of rational numbers

(C) set of irrational numbers

(D) set of nonzero real numbers

(E) set of real numbers

---

21. Let P_1 be the set of all primes, $\{2, 3, 5, 7, \ldots\}$, and for each integer n, let P_n be the set of all prime multiples of n, $\{2n, 3n, 5n, 7n, \ldots\}$. Which of the following intersections is nonempty?

(A) $P_1 \cap P_{23}$ (B) $P_7 \cap P_{21}$ (C) $P_{12} \cap P_{20}$ (D) $P_{20} \cap P_{24}$ (E) $P_5 \cap P_{25}$

---

22. Let $C(\mathbb{R})$ be the collection of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Then $C(\mathbb{R})$ is a real vector space with pointwise addition and scalar multiplication defined by

$$(f+g)(x) = f(x) + g(x) \text{ and } (rf)(x) = rf(x)$$

for all $f, g \in C(\mathbb{R})$ and all $r, x \in \mathbb{R}$. Which of the following are subspaces of $C(\mathbb{R})$?

I. $\{f : f \text{ is twice differentiable and } f''(x) - 2f'(x) + 3f(x) = 0 \text{ for all } x\}$

II. $\{g : g \text{ is twice differentiable and } g''(x) = 3g'(x) \text{ for all } x\}$

III. $\{h : h \text{ is twice differentiable and } h''(x) = h(x) + 1 \text{ for all } x\}$

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

---

23. For what value of b is the line $y = 10x$ tangent to the curve $y = e^{bx}$ at some point in the xy-plane?

(A) $\frac{10}{e}$ (B) 10 (C) $10e$ (D) e^{10} (E) e

---

24. Let h be the function defined by $h(x) = \int_0^{x^2} e^{x+t}\,dt$ for all real numbers x. Then $h'(1) =$

(A) $e - 1$ (B) e^2 (C) $e^2 - e$ (D) $2e^2$ (E) $3e^2 - e$

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25. Let $\{a_n\}_{n=1}^{\infty}$ be defined recursively by $a_1 = 1$ and $a_{n+1} = \left(\frac{n+2}{n}\right)a_n$ for $n \geq 1$. Then a_{30} is equal to

(A) $(15)(31)$ (B) $(30)(31)$ (C) $\frac{31}{29}$ (D) $\frac{32}{30}$ (E) $\frac{32!}{30!\,2!}$

---

26. Let $f(x, y) = x^2 - 2xy + y^3$ for all real x and y. Which of the following is true?

(A) f has all of its relative extrema on the line $x = y$.

(B) f has all of its relative extrema on the parabola $x = y^2$.

(C) f has a relative minimum at $(0, 0)$.

(D) f has an absolute minimum at $\left(\frac{2}{3}, \frac{2}{3}\right)$.

(E) f has an absolute minimum at $(1, 1)$.

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27. Consider the two planes $x + 3y - 2z = 7$ and $2x + y - 3z = 0$ in $\mathbb{R}^3$. Which of the following sets is the intersection of these planes?

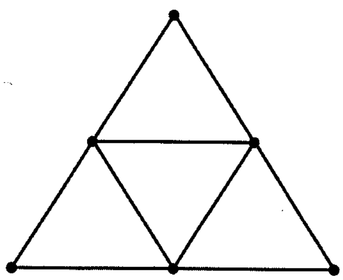
(A) $\varnothing$

(B) $\{(0, 3, 1)\}$

(C) $\{(x, y, z) : x = t,\ y = 3t,\ z = 7 - 2t,\ t \in \mathbb{R}\}$

(D) $\{(x, y, z) : x = 7t,\ y = 3 + t,\ z = 1 + 5t,\ t \in \mathbb{R}\}$

(E) $\{(x, y, z) : x - 2y - z = -7\}$



28. The figure above shows an undirected graph with six vertices. Enough edges are to be deleted from the graph in order to leave a spanning tree, which is a connected subgraph having the same six vertices and no cycles. How many edges must be deleted?

(A) One (B) Two (C) Three (D) Four (E) Five

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29. For all positive functions f and g of the real variable x, let $\sim$ be a relation defined by

$$f \sim g \text{ if and only if } \lim_{x \to \infty} \frac{f(x)}{g(x)} = 1.$$

Which of the following is NOT a consequence of $f \sim g$?

(A) $f^2 \sim g^2$ (B) $\sqrt{f} \sim \sqrt{g}$ (C) $e^f \sim e^g$ (D) $f + g \sim 2g$ (E) $g \sim f$

---

30. Let f be a function from a set X to a set Y. Consider the following statements.

P: For each $x \in X$, there exists $y \in Y$ such that $f(x) = y$.
Q: For each $y \in Y$, there exists $x \in X$ such that $f(x) = y$.
R: There exist $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $f(x_1) = f(x_2)$.

The negation of the statement "f is one-to-one and onto Y" is

(A) P or not R

(B) R or not P

(C) R or not Q

(D) P and not R

(E) R and not Q

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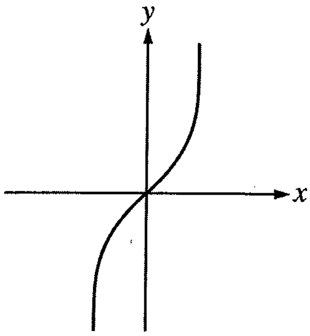
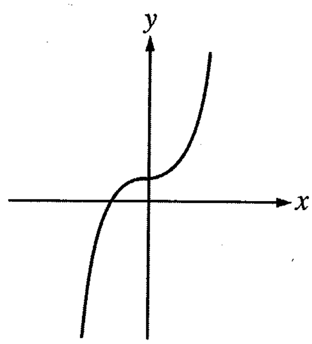
31. Which of the following most closely represents the graph of a solution to the differential equation $\frac{dy}{dx} = 1 + y^4$

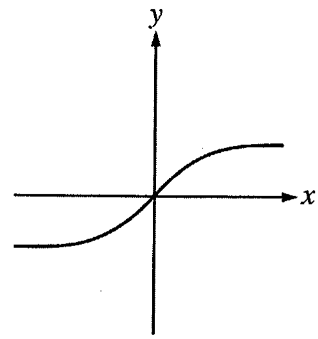
(A)



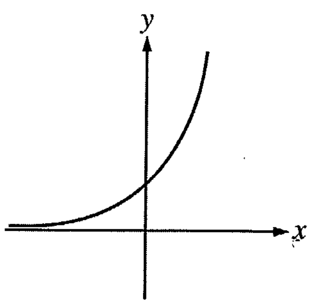
(B)



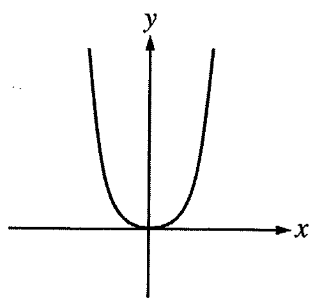
(C)



(D)



(E)



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32. Suppose that two binary operations, denoted by $\oplus$ and $\odot$, are defined on a nonempty set S, and that the following conditions are satisfied for all x, y, and z in S:

(1) $x \oplus y$ and $x \odot y$ are in S.
(2) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$ and $x \odot (y \odot z) = (x \odot y) \odot z$.
(3) $x \oplus y = y \oplus x$

Also, for each x in S and for each positive integer n, the elements nx and x^n are defined recursively as follows:

$1x = x^1 = x$ and

if kx and x^k have been defined, then $(k+1)x = kx \oplus x$ and $x^{k+1} = x^k \odot x$.

Which of the following must be true?

I. $(x \odot y)^n = x^n \odot y^n$ for all x and y in S and for each positive integer n.

II. $n(x \oplus y) = nx \oplus ny$ for all x and y in S and for each positive integer n.

III. $x^m \odot x^n = x^{m+n}$ for each x in S and for all positive integers m and n.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

33. The Euclidean algorithm is used to find the greatest common divisor (`gcd`) of two positive integers `a` and `b`.

```
input(a)
input(b)
while b > 0
  begin
    r := a mod b
    a := b
    b := r
  end
gcd := a
output(gcd)
```

When the algorithm is used to find the greatest common divisor of `a = 273` and `b = 110`, which of the following is the sequence of computed values for `r` ?

(A) `2, 26, 1, 0`

(B) `2, 53, 1, 0`

(C) `53, 2, 1, 0`

(D) `53, 4, 1, 0`

(E) `53, 5, 1, 0`

---

34. The minimal distance between any point on the sphere $(x-2)^2+(y-1)^2+(z-3)^2=1$ and any point on the sphere $(x+3)^2+(y-2)^2+(z-4)^2=4$ is

(A) 0 (B) 4 (C) $\sqrt{27}$ (D) $2(\sqrt{2}+1)$ (E) $3(\sqrt{3}-1)$

GO ON TO THE NEXT PAGE.

35. At a banquet, 9 women and 6 men are to be seated in a row of 15 chairs. If the entire seating arrangement is to be chosen at random, what is the probability that all of the men will be seated next to each other in 6 consecutive positions?

(A) $\dfrac{1}{\binom{15}{6}}$ (B) $\dfrac{6!}{\binom{15}{6}}$ (C) $\dfrac{10!}{15!}$ (D) $\dfrac{6!9!}{14!}$ (E) $\dfrac{6!10!}{15!}$

---

36. Let M be a 5×5 real matrix. Exactly four of the following five conditions on M are equivalent to each other. Which of the five conditions is equivalent to NONE of the other four?

(A) For any two distinct column vectors $\mathbf{u}$ and $\mathbf{v}$ of M, the set $\{\mathbf{u}, \mathbf{v}\}$ is linearly independent.

(B) The homogeneous system $M\mathbf{x} = \mathbf{0}$ has only the trivial solution.

(C) The system of equations $M\mathbf{x} = \mathbf{b}$ has a unique solution for each real 5×1 column vector $\mathbf{b}$.

(D) The determinant of M is nonzero.

(E) There exists a 5×5 real matrix N such that NM is the 5×5 identity matrix.

---

37. In the complex z-plane, the set of points satisfying the equation $z^2 = |z|^2$ is a

(A) pair of points

(B) circle

(C) half-line

(D) line

(E) union of infinitely many different lines

GO ON TO THE NEXT PAGE.

38. Let A and B be nonempty subsets of $\mathbb{R}$ and let $f : A \to B$ be a function. If $C \subseteq A$ and $D \subseteq B$, which of the following must be true?

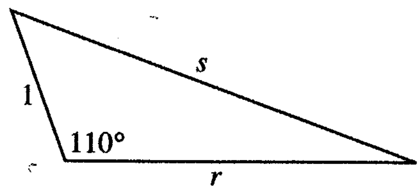
(A) $C \subseteq f^{-1}\left(f(C)\right)$

(B) $D \subseteq f\left(f^{-1}(D)\right)$

(C) $f^{-1}\left(f(C)\right) \subseteq C$

(D) $f^{-1}\left(f(C)\right) = f\left(f^{-1}(D)\right)$

(E) $f\left(f^{-1}(D)\right) = f^{-1}(D)$



39. In the figure above, as r and s increase, the length of the third side of the triangle remains 1 and the measure of the obtuse angle remains 110°. What is $\lim_{\substack{s \to \infty \\ r \to \infty}} (s - r)$?

(A) 0

(B) A positive number less than 1

(C) 1

(D) A finite number greater than 1

(E) ∞

GO ON TO THE NEXT PAGE.

40. For which of the following rings is it possible for the product of two nonzero elements to be zero?

(A) The ring of complex numbers

(B) The ring of integers modulo 11

(C) The ring of continuous real-valued functions on $[0, 1]$

(D) The ring $\{a + b\sqrt{2} : a \text{ and } b \text{ are rational numbers}\}$

(E) The ring of polynomials in x with real coefficients

---

41. Let C be the circle $x^2 + y^2 = 1$ oriented counterclockwise in the xy-plane. What is the value of the line integral $\oint_C (2x - y)\,dx + (x + 3y)\,dy$?

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) π (E) 2π

---

42. Suppose X is a discrete random variable on the set of positive integers such that for each positive integer n, the probability that $X = n$ is $\frac{1}{2^n}$. If Y is a random variable with the same probability distribution and X and Y are independent, what is the probability that the value of at least one of the variables X and Y is greater than 3 ?

(A) $\frac{1}{64}$ (B) $\frac{15}{64}$ (C) $\frac{1}{4}$ (D) $\frac{3}{8}$ (E) $\frac{4}{9}$

GO ON TO THE NEXT PAGE.

43. If $z = e^{2\pi i/5}$, then $1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9 =$

(A) 0 (B) $4e^{3\pi i/5}$ (C) $5e^{4\pi i/5}$ (D) $-4e^{2\pi i/5}$ (E) $-5e^{3\pi i/5}$

---

44. A fair coin is to be tossed 100 times, with each toss resulting in a head or a tail. If H is the total number of heads and T is the total number of tails, which of the following events has the greatest probability?

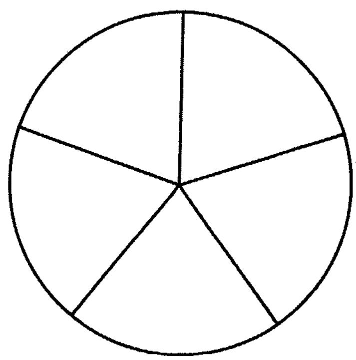
(A) $H = 50$

(B) $T \geq 60$

(C) $51 \leq H \leq 55$

(D) $H \geq 48$ and $T \geq 48$

(E) $H \leq 5$ or $H \geq 95$



45. A circular region is divided by 5 radii into sectors as shown above. Twenty-one points are chosen in the circular region, none of which is on any of the 5 radii. Which of the following statements must be true?

I. Some sector contains at least 5 of the points.

II. Some sector contains at most 3 of the points.

III. Some pair of adjacent sectors contains a total of at least 9 of the points.

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

GO ON TO THE NEXT PAGE.

46. Let G be the group of complex numbers $\{1, i, -1, -i\}$ under multiplication. Which of the following statements are true about the homomorphisms of G into itself?

I. $z \mapsto \overline{z}$ defines one such homomorphism, where $\overline{z}$ denotes the complex conjugate of z.

II. $z \mapsto z^2$ defines one such homomorphism.

III. For every such homomorphism, there is an integer k such that the homomorphism has the form $z \mapsto z^k$.

(A) None (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

47. Let $\mathbf{F}$ be a constant unit force that is parallel to the vector $(-1, 0, 1)$ in xyz-space. What is the work done by $\mathbf{F}$ on a particle that moves along the path given by (t, t^2, t^3) between time $t = 0$ and time $t = 1$?

(A) $-\frac{1}{4}$ (B) $-\frac{1}{4\sqrt{2}}$ (C) 0 (D) $\sqrt{2}$ (E) $3\sqrt{2}$

---

48. Consider the theorem: If f and f' are both strictly increasing real-valued functions on the interval $(0, \infty)$, then $\lim_{x\to\infty} f(x) = \infty$. The following argument is suggested as a proof of this theorem.

(1) By the Mean Value Theorem, there is a c_1 in the interval $(1, 2)$ such that

$$f'(c_1) = \frac{f(2) - f(1)}{2 - 1} = f(2) - f(1) > 0.$$

(2) For each $x > 2$, there is a c_x in $(2, x)$ such that $\frac{f(x) - f(2)}{x - 2} = f'(c_x)$.

(3) For each $x > 2$, $\frac{f(x) - f(2)}{x - 2} = f'(c_x) > f'(c_1)$ since f' is strictly increasing.

(4) For each $x > 2$, $f(x) > f(2) + (x - 2)f'(c_1)$.

(5) $\lim_{x\to\infty} f(x) = \infty$

Which of the following statements is true?

(A) The argument is valid.

(B) The argument is not valid since the hypotheses of the Mean Value Theorem are not satisfied in (1) and (2).

(C) The argument is not valid since (3) is not valid.

(D) The argument is not valid since (4) cannot be deduced from the previous steps.

(E) The argument is not valid since (4) does not imply (5).

---

49. Up to isomorphism, how many additive abelian groups G of order 16 have the property that $x + x + x + x = 0$ for each x in G?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

---

50. Let A be a real 2×2 matrix. Which of the following statements must be true?

I. All of the entries of A^2 are nonnegative.

II. The determinant of A^2 is nonnegative.

III. If A has two distinct eigenvalues, then A^2 has two distinct eigenvalues.

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

---

51. If $\lfloor x \rfloor$ denotes the greatest integer not exceeding x, then $\int_0^{\infty} \lfloor x \rfloor e^{-x}\, dx =$

(A) $\frac{e}{e^2 - 1}$ (B) $\frac{1}{e - 1}$ (C) $\frac{e - 1}{e}$ (D) 1 (E) $+\infty$

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52. If A is a subset of the real line $\mathbb{R}$ and A contains each rational number, which of the following must be true?

(A) If A is open, then $A = \mathbb{R}$.

(B) If A is closed, then $A = \mathbb{R}$.

(C) If A is uncountable, then $A = \mathbb{R}$.

(D) If A is uncountable, then A is open.

(E) If A is countable, then A is closed.

---

53. What is the minimum value of the expression $x + 4z$ as a function defined on $\mathbb{R}^3$, subject to the constraint $x^2 + y^2 + z^2 \leq 2$?

(A) 0 (B) -2 (C) $-\sqrt{34}$ (D) $-\sqrt{35}$ (E) $-5\sqrt{2}$

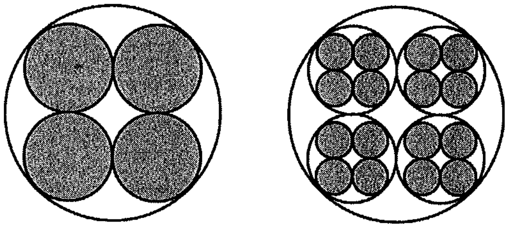

Figure 1 Figure 2

54. The four shaded circles in Figure 1 above are congruent and each is tangent to the large circle and to two of the other shaded circles. Figure 2 is the result of replacing each of the shaded circles in Figure 1 by a figure that is geometrically similar to Figure 1. What is the ratio of the area of the shaded portion of Figure 2 to the area of the shaded portion of Figure 1 ?

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{1+\sqrt{2}}$ (C) $\frac{4}{1+\sqrt{2}}$ (D) $\left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)^2$ (E) $\left(\frac{2}{1+\sqrt{2}}\right)^2$

GO ON TO THE NEXT PAGE.

55. For how many positive integers k does the ordinary decimal representation of the integer $k!$ end in exactly 99 zeros?

(A) None (B) One (C) Four (D) Five (E) Twenty-four

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56. Which of the following does NOT define a metric on the set of all real numbers?

(A) $\delta(x, y) = \begin{cases} 0 & \text{if } x = y \\ 2 & \text{if } x \neq y \end{cases}$

(B) $\rho(x, y) = \min\{|x - y|, 1\}$

(C) $\sigma(x, y) = \dfrac{|x - y|}{3}$

(D) $\tau(x, y) = \dfrac{|x - y|}{|x - y| + 1}$

(E) $\omega(x, y) = (x - y)^2$

---

57. The set of real numbers x for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{n!x^{2n}}{n^n\left(1 + x^{2n}\right)}$ converges is

(A) $\{0\}$

(B) $\{x : -1 < x < 1\}$

(C) $\{x : -1 \leq x \leq 1\}$

(D) $\{x : -\sqrt{e} \leq x \leq \sqrt{e}\}$

(E) $\mathbb{R}$

GO ON TO THE NEXT PAGE.

58. Suppose A and B are $n \times n$ invertible matrices, where $n > 1$, and I is the $n \times n$ identity matrix. If A and B are similar matrices, which of the following statements must be true?

I. $A - 2I$ and $B - 2I$ are similar matrices.

II. A and B have the same trace.

III. A^{-1} and B^{-1} are similar matrices.

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

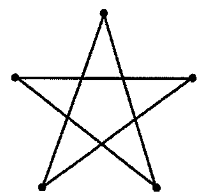
---

59. Suppose f is an analytic function of the complex variable $z = x + iy$ given by

$$f(z) = (2x + 3y) + ig(x, y),$$

where $g(x, y)$ is a real-valued function of the real variables x and y. If $g(2, 3) = 1$, then $g(7, 3) =$

(A) -14 (B) -9 (C) 0 (D) 11 (E) 18



60. The group of symmetries of the regular pentagram shown above is isomorphic to the

(A) symmetric group S_5

(B) alternating group A_5

(C) cyclic group of order 5

(D) cyclic group of order 10

(E) dihedral group of order 10

GO ON TO THE NEXT PAGE.

61. Which of the following sets has the greatest cardinality?

(A) $\mathbb{R}$

(B) The set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$

(C) The set of all functions from $\mathbb{R}$ to $\{0, 1\}$

(D) The set of all finite subsets of $\mathbb{R}$

(E) The set of all polynomials with coefficients in $\mathbb{R}$

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62. Let K be a nonempty subset of $\mathbb{R}^n$, where $n > 1$. Which of the following statements must be true?

I. If K is compact, then every continuous real-valued function defined on K is bounded.

II. If every continuous real-valued function defined on K is bounded, then K is compact.

III. If K is compact, then K is connected.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

---

63. If f is the function defined by

$$f(x) = \begin{cases} xe^{-x^2-x^{-2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases}$$

at how many values of x does the graph of f have a horizontal tangent line?

(A) None (B) One (C) Two (D) Three (E) Four

---

64. For each positive integer n, let f_n be the function defined on the interval $[0, 1]$ by $f_n(x) = \dfrac{x^n}{1 + x^n}$. Which of the following statements are true?

I. The sequence $\{f_n\}$ converges pointwise on $[0, 1]$ to a limit function f.

II. The sequence $\{f_n\}$ converges uniformly on $[0, 1]$ to a limit function f.

III. $\lim\limits_{n\to\infty} \int_0^1 f_n(x)\,dx = \int_0^1 \left(\lim\limits_{n\to\infty} f_n(x)\right) dx$

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

---

65. Which of the following statements are true about the open interval $(0, 1)$ and the closed interval $[0, 1]$?

I. There is a continuous function from $(0, 1)$ onto $[0, 1]$.

II. There is a continuous function from $[0, 1]$ onto $(0, 1)$.

III. There is a continuous one-to-one function from $(0, 1)$ onto $[0, 1]$.

(A) None (B) I only (C) II only (D) I and III only (E) I, II, and III

GO ON TO THE NEXT PAGE.

66. Let R be a ring with a multiplicative identity. If U is an additive subgroup of R such that $ur \in U$ for all $u \in U$ and for all $r \in R$, then U is said to be a right ideal of R. If R has exactly two right ideals, which of the following must be true?

I. R is commutative.

II. R is a division ring (that is, all elements except the additive identity have multiplicative inverses).

III. R is infinite.

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

STOP

If you finish before time is called, you may check your work on this test.

GO ON TO THE NEXT PAGE.

Worksheet for the GRE Mathematics Test, Form GR0568
Answer Key and Percentages* of Examinees Answering Each Question Correctly

QUESTION Number	Answer	P+	RESPONSE C	I
1	B	84		
2	E	84		
3	D	83		
4	B	74		
5	B	95		
6	C	73		
7	C	78		
8	A	73		
9	A	62		
10	B	84		
11	E	56		
12	C	57		
13	B	60		
14	D	75		
15	E	68		
16	D	47		
17	B	63		
18	A	54		
19	C	61		
20	D	61		
21	C	74		
22	B	51		
23	A	49		
24	E	50		
25	A	60		
26	A	39		
27	D	66		
28	D	64		
29	C	52		
30	C	62		
31	A	55		
32	D	56		
33	D	88		
34	E	52		
35	E	52		

QUESTION Number	Answer	P+	RESPONSE C	I
36	A	47		
37	D	52		
38	A	43		
39	B	42		
40	C	48		
41	E	53		
42	B	48		
43	E	26		
44	D	41		
45	D	68		
46	E	42		
47	C	28		
48	A	37		
49	D	33		
50	B	34		
51	B	30		
52	B	35		
53	C	29		
54	E	25		
55	D	28		
56	E	38		
57	E	30		
58	E	26		
59	A	29		
60	E	43		
61	C	36		
62	D	34		
63	D	14		
64	D	36		
65	B	35		
66	B	42		

Total Correct (C) ____________

Total Incorrect (I) ____________

Total Score:

C – I/4 = ________

Scaled Score (SS) = ________

* The P+ column indicates the percent of Mathematics Test examinees who answered each question correctly; it is based on a sample of December 2005 examinees selected to represent all Mathematics Test examinees tested between July 1, 2004, and June 30, 2007.

Evaluating Your Performance

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test. Both the worksheet on page 66 and the table on page 67 use performance data from GRE Mathematics Test examinees.

The data in the worksheet on page 66 are based on the performance of a sample of the examinees who took this test in December 2005. This sample was selected to represent the total population of GRE Mathematics Test examinees tested between July 2004 and June 2007. The numbers in the column labeled "P+" on the worksheet indicate the percentages of examinees in this sample who answered each question correctly. You may use these numbers as a guide for evaluating your performance on each test question.

The table on page 67 contains, for each scaled score, the percentage of examinees tested between July 2004 and June 2007 who received lower scores. Interpretive data based on the scores earned by examinees tested in this three-year period will be used by admissions officers in the 2008-09 testing year. These percentages appear in the score conversion table in a column to the right of the scaled scores. For example, in the percentage column opposite the scaled score of 640 is the number 52. This means that 52 percent of the GRE Mathematics Test examinees tested between July 2004 and June 2007 scored lower than 640. To compare yourself with this population, look at the percentage next to the scaled score you earned on the practice test.

It is important to realize that the conditions under which you tested yourself were not exactly the same as those you will encounter at a test center. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on this practice test with the performance of other GRE Mathematics Test examinees, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test under standard conditions.