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Recall $Sp^G = [\mathcal{J}_G, \mathcal{T}^G]$

where $\mathcal{J}_G$ is the Mandell-May cat and $\mathcal{T}^G$ is pointed G-spaces

A map $f: X \to Y$ in $\mathcal{T}^G$ is a weq or fib if $f^H: X^H \to Y^H$ is one for each $H \subseteq G$. It has cofib gen sets

$I_G = \{ G \underset{H}{\times} (S^{n-1} \to D^n)_+ : n \geq 0, H \subseteq G \}$

and $J_G = \{ G \underset{H}{\times} (I^n \to I^{n+1})_+ : \quad " \quad \}$

~~The projective model~~

Def. Let $\mathcal{M}$ be a CGMC with gen sets I and J and let $\mathcal{J}$ be a small cat (enriched over $\mathcal{M}$). Then $[\mathcal{J}, \mathcal{M}]$ has a <u>~~model~~ projective structure</u> where

$f: X \to Y$ is a weq or fib iff

$f_j: X_j \to Y_j$ is one $\forall j \in \mathcal{J}$

It has gen sets

$$I_{\mathcal{J}} = \{ S^V \times I : V \in \mathcal{J} \}$$
$$J_{\mathcal{J}} = \{ S^V \times J : V \in \mathcal{J} \}$$

<u>Positivization</u> STATE KAN TRANSFER THM.

~~Suppose~~ $\mathcal{J}$ has a full sub cat $\mathcal{L}$

with $\mathcal{L} \xhookrightarrow{\lambda} \mathcal{J}$

$$[\mathcal{L}, \mathcal{M}] \underset{\lambda_!}{\overset{\lambda^*}{\underset{\top}{\rightleftarrows}}} [\mathcal{J}, \mathcal{M}].$$

Using the Kan transfer thm we get an <u>induced model structure</u> on $[\mathcal{J}, \mathcal{M}]$ in which $f: X \to Y$ is a weq or fib if $f_\ell : X_\ell \to Y_\ell$ is one for each $\ell \in \mathcal{L}_0$

Case of interest: Let $\mathcal{L}_G \subset \mathcal{J}_G$ be the full subcat of reps V with $V^G \neq 0$ (positive ideal)

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Stabilization is a form of Bousfield localization. Given a model cat M ~~we form a new model~~ and a set (or class) of ~~morphisms~~ (cofibs?) $\underline{S}$, we form a new model cat $L_S M$ where

1) the underlying category is the same
2) It has the same cofibs
3) ~~Each~~ Each weq in M and each map in $\underline{S}$ is a weq in $L_{\underline{S}} M$

This means $L_S M$ has more trivial cofibs and hence fewer fibs and more interesting fibrant replacement functor

Example. In the classical case the stably fibrant objects are the Ω-spectra.

We have its generating set of triv cof.

Describe Σ in equiv case

and positive equiv case
if time permits.

Enlargement

~~Then~~ are gen sets for Sp^G (proj)

1. $\{S^{-V} \wedge I_G : V^G \neq 0\}$ or $\{S^{-V} \wedge I_G\}$

We need a bigger set

2. $\{G_+ \underset{H}{\wedge} S^{-V} \wedge I_H : H \leq G,\ V^H \neq 0\}$

in order for the change of gp adjunction

$$Sp^H \underset{i^G_H}{\overset{G_+ \wedge_H (-)}{\rightleftarrows}} Sp^G \quad (\perp)$$

to be a Quillen adjunction $\forall H$

Reformulation

$$\prod_{H \leq G} Sp^H \underset{\prod i^G_H}{\overset{\prod G_+ \wedge_H (-)}{\rightleftarrows}} \prod_{H \leq G} Sp^G \underset{\Delta}{\overset{\bigvee}{\rightleftarrows}} Sp^G$$

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NEW Thm (Enlargement)

Let $\mathcal{M}$ and $\mathcal{M}'$ be CGMCs with gen sets (I, J) and (I', J')

Suppose we have an adjunction

(not Quillen) $\mathcal{M}' \underset{U}{\overset{F}{\underset{\perp}{\rightleftarrows}}} \mathcal{M}$

~~with UFJ'~~ where U preserve weq and UFJ' is set of weqs in $\mathcal{M}'$

Then consider the enlargement adj

$$\mathcal{M} \times \mathcal{M}' \underset{\mathcal{M} \times U}{\overset{\mathcal{M} \times F}{\underset{\perp}{\rightleftarrows}}} \mathcal{M} \times \mathcal{M} \underset{\Delta}{\overset{V}{\underset{\perp}{\rightleftarrows}}} \mathcal{M}$$

Gives us a model structure on $\mathcal{M}$ with same weqs

generating sets $(I \cup FI', \cancel{x}\, J \cup FJ')$

(more cofibs) and fibrations with suitable ~~left~~ RLP.

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Example $\mathcal{M} = Sp^{G}$

$\mathcal{M}' = Sp^{H}$

with change of gp adjunction