Algebraic Topology, in memory of Hans-Joachim Baues

The Hill-Lawson spectral sequence and the telescope conjecture

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The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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This is joint work with Mike Hill and Tyler Lawson.

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The MRS spectrum $y(m)$
Inverting v_m
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The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
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Internal Steenrod operations for $y(m)$
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A possible $\mathbb{E}_2$ structure
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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
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Internal Steenrod operations for $y(m)$
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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

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The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

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Here the superscripts are topological dimension and filtration, the (x, y) convention. For $p = 2$, there is a similar description with $b_{i,j} = h_{i,j}^2$.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
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Here the superscripts are topological dimension and filtration, the (x, y) convention. For $p = 2$, there is a similar description with $b_{i,j} = h_{i,j}^2$. In general it is a p-fold Massey product.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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Again, for the Adams spectral sequence,

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
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A possible $\mathbb{E}_2$ structure
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Note that the Hill-Lawson filtration is higher than that of Adams.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The MRS spectrum $y(m)$

Doug Ravenel

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The MRS spectrum $y(m)$
Inverting v_m
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Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

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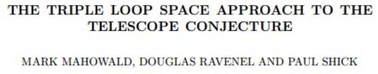

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
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Internal Steenrod operations for $y(m)$
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The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

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The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

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Internal Steenrod operations for $y(m)$

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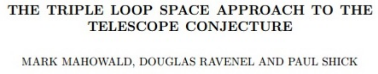
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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The MRS spectrum $y(m)$ (continued)

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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

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For a prime p and positive integer m, let $y(m)$ denote the Thom spectum of the restriction of λ_p induced by the map

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Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

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This spectrum has some pleasant properties. Recall that

$$H_*H/p \cong E(\tau_0, \tau_1, \ldots) \otimes P(\xi_1, \xi_2, \ldots)$$
$$\text{with } |\tau_i| = 2p^i - 1 \text{ and } |\xi_i| = 2p^i - 2.$$

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
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A possible $\mathbb{E}_2$ structure
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It turns out that

$$H_* y(m) \cong E(\tau_0, \dots, \tau_{m-1}) \otimes P(\xi_1, \dots, \xi_m).$$

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
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Internal Steenrod operations for $y(m)$
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This implies that the Adams spectral sequence for $y(m)$ has

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Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
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A possible $\mathbb{E}_2$ structure
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We have added m to each (first) subscript.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

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Internal Steenrod operations for $y(m)$

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A possible $\mathbb{E}_2$ structure

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We have added m to each (first) subscript. There is a Hill-Lawson spectral sequence having a similar E_1-term in which each element has filtration divisible by p^m.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
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Internal Steenrod operations for $y(m)$
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A possible $\mathbb{E}_2$ structure
Conclusion

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We can invert v_m on the spectrum level as follows.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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$$\Sigma\Omega S^3 \simeq \bigvee_{k>0} S^{2k+1},$$

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m

We can invert v_m on the spectrum level as follows. James showed that

$$\Sigma\Omega S^3 \simeq \bigvee_{k>0} S^{2k+1},$$

so for each $k > 0$ we get a James-Hopf map

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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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When $k = p^m$, its p-local homotopy theoretic fiber is our friend $J_{p^m-1}S^2$.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1*s*
A possible $\mathbb{E}_2$ *structure*
Conclusion

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When $k = p^m$, its p-local homotopy theoretic fiber is our friend $J_{p^m-1}S^2$. It follows that there is a fiber sequence

$$\Omega^3 S^{2p^m+1} \to \Omega J_{p^m-1} S^2 \to \Omega^2 S^3,$$

which Thomifies to

$$\Sigma^\infty \Omega^3 S^{2p^m+1} \to y(m) \to H/p.$$

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m (continued)

The composite

$$S^{2p^m-2} \to \Sigma^\infty \Omega^3 S^{2p^m+1} \to y(m)$$

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

Inverting v_m (continued)

The composite

$$S^{2p^m-2} \to \Sigma^\infty \Omega^3 S^{2p^m+1} \to y(m)$$

leads to a self map

$$\Sigma^{2p^m-2} y(m) \xrightarrow{v_m} y(m).$$

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m (continued)

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It is known to by induce an isomorphism in Morava K-theory $K(m)_*$.

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

Inverting v_m (continued)

The composite

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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

Inverting v_m (continued)

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It is known to by induce an isomorphism in Morava K-theory $K(m)_*$. We can iterate it to form a telescope, the homotopy colimit of

$$y(m) \xrightarrow{v_m} \Sigma^{-|v_m|} y(m) \xrightarrow{v_m} \Sigma^{-2|v_m|} y(m) \longrightarrow \dots,$$

which we denote by $Y(m)$.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m (continued)

The telescope $Y(m)$ is the homotopy colimit of

$$y(m) \xrightarrow{v_m} \Sigma^{-|v_m|} y(m) \xrightarrow{v_m} \Sigma^{-2|v_m|} y(m) \longrightarrow \cdots .$$

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m (continued)

The telescope $Y(m)$ is the homotopy colimit of

$$y(m) \xrightarrow{v_m} \Sigma^{-|v_m|} y(m) \xrightarrow{v_m} \Sigma^{-2|v_m|} y(m) \longrightarrow \dots .$$

It admits a map to $L_{K(m)} y(m)$,

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m (continued)

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It admits a map to $L_{K(m)} y(m)$, which the height m form of the telescope conjecture says is an equivalence.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m (continued)

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$$y(m) \xrightarrow{v_m} \Sigma^{-|v_m|} y(m) \xrightarrow{v_m} \Sigma^{-2|v_m|} y(m) \longrightarrow \dots.$$

It admits a map to $L_{K(m)} y(m)$, which the height m form of the telescope conjecture says is an equivalence. Thus showing $Y(m)$ and $L_{K(m)} y(m)$ are diffferent for $m > 1$ would disprove the telescope conjecture.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

Inverting v_m (continued)

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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

Inverting v_m (continued)

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There is a well understood Adams-Novikov spectral sequence converging to $\pi_* L_{K(m)} y(m)$.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

Inverting v_m (continued)

The telescope $Y(m)$ is the homotopy colimit of

$$y(m) \xrightarrow{v_m} \Sigma^{-|v_m|} y(m) \xrightarrow{v_m} \Sigma^{-2|v_m|} y(m) \longrightarrow \cdots .$$

It admits a map to $L_{K(m)} y(m)$, which the height m form of the telescope conjecture says is an equivalence. Thus showing $Y(m)$ and $L_{K(m)} y(m)$ are diffferent for $m > 1$ would disprove the telescope conjecture. They are known to be the same for $m = 1$.

There is a well understood Adams-Novikov spectral sequence converging to $\pi_* L_{K(m)} y(m)$.

There are localized forms of both the Adams and Hill-Lawson spectral sequences that converge to $\pi_* Y(m)$.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Inverting v_m (continued)

The telescope $Y(m)$ is the homotopy colimit of

$$y(m) \xrightarrow{v_m} \Sigma^{-|v_m|} y(m) \xrightarrow{v_m} \Sigma^{-2|v_m|} y(m) \longrightarrow \dots .$$

It admits a map to $L_{K(m)} y(m)$, which the height m form of the telescope conjecture says is an equivalence. Thus showing $Y(m)$ and $L_{K(m)} y(m)$ are diffferent for $m > 1$ would disprove the telescope conjecture. They are known to be the same for $m = 1$.

There is a well understood Adams-Novikov spectral sequence converging to $\pi_* L_{K(m)} y(m)$.

There are localized forms of both the Adams and Hill-Lawson spectral sequences that converge to $\pi_* Y(m)$. The latter is a new tool for studying the telescope conjecture.

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The Adams-Novikov spectral sequence

The Adams-Novikov E_2-term for $L_{K(m)}y(m)$ is

$$R_m \otimes E(h_{m+i,j} : 1 \leq i, j+1 \leq m),$$
$$\text{where } R_m = v_m^{-1} P(v_m, \ldots, v_{2m}).$$

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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This is an exterior algebra on m^2 odd dimensional generators tensored with an even dimensional localized polynomial ring.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The Adams-Novikov spectral sequence

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The exterior algebra is the cohomology of a certain open subgroup of the mth Morava stabilizer group.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1s

A possible $\mathbb{E}_2$ structure

Conclusion

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This is an exterior algebra on m^2 odd dimensional generators tensored with an even dimensional localized polynomial ring. Each v_{m+i} has filtration 0, and each $h_{m+i,j}$ has filtration 1. The spectral sequence collapses for large primes.

The exterior algebra is the cohomology of a certain open subgroup of the mth Morava stabilizer group. It is cofinite with index $p^{m^2-m}(p^m - 1)$.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Adams spectral sequence

To repeat, the Adams-Novikov E_2-term for $L_{K(m)}y(m)$ is

$$R_m \otimes E(h_{h+i,j} : 1 \leq i, j+1 \leq h).$$

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1s

A possible $\mathbb{E}_2$ structure

Conclusion

The localized Adams spectral sequence

To repeat, the Adams-Novikov E_2-term for $L_{K(m)}y(m)$ is

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The localized Adams E_2-term for $Y(m)$ is

$$R_m \otimes E(h_{m+i,j}) \otimes P(b_{m+i,j})$$
$$\text{where } i > 0 \text{ and } 0 \le j \le m-1$$
$$\text{and } R_m = v_m^{-1} P(v_m, \ldots, v_{2m}).$$

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The localized Adams spectral sequence

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$$\text{and } R_m = v_m^{-1} P(v_m, \ldots, v_{2m}).$$

The Adams filtration of each v_{m+i} is 1 instead of 0.

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The localized Adams spectral sequence

To repeat, the Adams-Novikov E_2-term for $L_{K(m)}y(m)$ is

$$R_m \otimes E(h_{h+i,j} : 1 \leq i, j+1 \leq h).$$

The localized Adams E_2-term for $Y(m)$ is

$$R_m \otimes E(h_{m+i,j}) \otimes P(b_{m+i,j})$$

where $i > 0$ and $0 \leq j \leq m-1$

and $R_m = v_m^{-1}P(v_m, \ldots, v_{2m})$.

The Adams filtration of each v_{m+i} is 1 instead of 0. Unlike the Adams-Novikov E_2-term, it is infinitely generated over R_m.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Adams spectral sequence (continued)

To repeat, the localized Adams E_2-term for $Y(m)$ is

$$R_m \otimes E(h_{m+i,j}) \otimes P(b_{m+i,j}).$$

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The localized Adams spectral sequence (continued)

To repeat, the localized Adams E_2-term for $Y(m)$ is

$$R_m \otimes E(h_{m+i,j}) \otimes P(b_{m+i,j}).$$

We conjectured that there are differentials

$$d_{2p^j} h_{m+i,j} = v_m b_{i+j,m-1-j}^{p^j} \qquad \text{for } 0 \le j \le m-1 \text{ and } i+j > m.$$

and no others.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Adams spectral sequence (continued)

To repeat, the localized Adams E_2-term for $Y(m)$ is

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$$d_{2p^j} h_{m+i,j} = v_m b_{i+j,m-1-j}^{p^j} \qquad \text{for } 0 \le j \le m-1 \text{ and } i+j > m.$$

and no others. This would leave

$$E_\infty = R_m \otimes E(h_{m+i,j} : i+j \le m) \otimes P(b_{m+i,j})/(b_{m+i,j}^{p^{m-1-j}}).$$

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Adams spectral sequence (continued)

The conjectured localized Adams E_∞-term is

$$R_m \otimes E(h_{m+i,j} : i + j \leq m) \otimes P(b_{m+i,j} : i > 0, 0 \leq j \leq m-2)/(b_{m+i,j}^{p^{m-1-j}}).$$

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Adams spectral sequence (continued)

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For $m = 1$ this reads $R_1 \otimes E(h_{2,0})$,

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Adams spectral sequence (continued)

The conjectured localized Adams E_∞-term is

$$\begin{aligned} R_m \otimes E(h_{m+i,j} : i + j \leq m) \\ \otimes P(b_{m+i,j} : i > 0, 0 \leq j \leq m-2)/(b_{m+i,j}^{p^{m-1-j}}). \end{aligned}$$

For $m = 1$ this reads $R_1 \otimes E(h_{2,0})$, which is also the Adams-Novikov E_2-term.

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The localized Adams spectral sequence (continued)

The conjectured localized Adams E_∞-term is

$$R_m \otimes E(h_{m+i,j} : i + j \leq m)$$
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For $m = 1$ this reads $R_1 \otimes E(h_{2,0})$, which is also the Adams-Novikov E_2-term.

For $m > 1$ the number of exterior generators is $(m^2 + m)/2$, which is fewer than the m^2 generators predicted by the telescope conjecture.

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The localized Adams spectral sequence (continued)

The conjectured localized Adams E_∞-term is

$$R_m \otimes E(h_{m+i,j} : i + j \leq m)$$
$$\otimes P(b_{m+i,j} : i > 0, 0 \leq j \leq m - 2)/(b_{m+i,j}^{p^{m-1-j}}).$$

For $m = 1$ this reads $R_1 \otimes E(h_{2,0})$, which is also the Adams-Novikov E_2-term.

For $m > 1$ the number of exterior generators is $(m^2 + m)/2$, which is fewer than the m^2 generators predicted by the telescope conjecture. For $m = 2$, the above reads

$$R_2 \otimes E(h_{3,0}, h_{3,1}, h_{4,0}) \otimes P(b_{2+i,0} : i > 0)/(b_{2+i,0}^p).$$

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Adams spectral sequence (continued)

The conjectured localized Adams E_∞-term is

$$R_m \otimes E(h_{m+i,j} : i + j \le m) \otimes P(b_{m+i,j} : i > 0, 0 \le j \le m-2)/(b_{m+i,j}^{p^{m-1-j}}).$$

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For $m > 1$ the number of exterior generators is $(m^2 + m)/2$, which is fewer than the m^2 generators predicted by the telescope conjecture. For $m = 2$, the above reads

$$R_2 \otimes E(h_{3,0}, h_{3,1}, h_{4,0}) \otimes P(b_{2+i,0} : i > 0)/(b_{2+i,0}^p).$$

Unfortunately we were unable to prove that the expected differentials all occur or that the $b_{m+i,j}$s are all permanent cycles.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The localized Hill-Lawson spectral sequence

The Hill-Lawson E_1-term for the spectrum $y(m)$ is

$$E_1 = P(v_{m+n} : n \geq 0) \otimes E(h_{m+i,j} : i > 0, j \geq 0) \otimes P(b_{m+i,j})$$

$$v_{m+n} \in E_1^{2p^{M+n}-2,p^n}, \quad h_{m+i,j} \in E_1^{2p^j(p^{m+i}-1)-1,p^{i+j}}$$

$$b_{m+i,j} \in E_1^{2p^{j+1}(p^{m+i}-1)-2,p^{i+j+1}}.$$

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

The localized Hill-Lawson spectral sequence

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As we did for the (May) Adams E_1-term, we added m to all of the subscripts.

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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As we did for the (May) Adams E_1-term, we added m to all of the subscripts. Here we have divided the previously defined filtrations by p^m.

Before discussing differentials we need to describe some internal structure of $y(m)$.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1s
A possible $\mathbb{E}_2$ structure
Conclusion

Internal Steenrod operations for $y(m)$

Recall that

$$H_* y(m) \cong E(\tau_0, \ldots, \tau_{m-1}) \otimes P(\xi_1, \ldots, \xi_m) \subseteq \mathcal{A}_*,$$

where $\mathcal{A}_*$ is the dual Steenrod algebra.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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$$y(m) \wedge y(m) \simeq \bigvee_{\alpha} \Sigma^{|\alpha|} y(m)$$

with one summand for each monomial α in $H_* y(m)$,

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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$$y(m) \wedge y(m) \simeq \bigvee_{\alpha} \Sigma^{|\alpha|} y(m)$$

with one summand for each monomial α in $H_*y(m)$, and to maps (cohomology operations)

$$y(m) \xrightarrow{\theta^\alpha} \Sigma^{|\alpha|} y(m).$$

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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These lead to right actions of a certain quotient of the Steenrod algebra (the dual of $H_*y(m)$)

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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$$y(m) \xrightarrow{\theta^{\alpha}} \Sigma^{|\alpha|} y(m).$$

These lead to right actions of a certain quotient of the Steenrod algebra (the dual of $H_* y(m)$) on each of our spectral sequences.

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Internal Steenrod operations for $y(m)$ (continued)

The action of internal Steenrod operations in the Hill-Lawson E_1-term for each $i > 0$ is shown below.

$$
\begin{array}{ccccccc}
v_{m+i} & \xrightarrow{P^1} & v_{m+i-1}^{p} & \xrightarrow{P^p} & v_{m+i-2}^{p^2} & \xrightarrow{P^{p^2}} & \cdots \\
\Big\downarrow{\scriptstyle \beta} & & & & & & \\
h_{m+i,0} & \xrightarrow{P^1} & h_{m+i-1,1} & \xrightarrow{P^p} & h_{m+i-2,2} & \xrightarrow{P^{p^2}} & \cdots
\end{array}
$$

$$
h_{m+i,m} \xrightarrow{\beta} b_{m+i,m-1} \xrightarrow{P^1} b_{m+i,m-2}^{p} \xrightarrow{P^p} b_{m+i,m-3}^{p^2} \xrightarrow{P^{p^2}} \cdots
$$

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Internal Steenrod operations for $y(m)$ (continued)

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 & & \downarrow \beta & & & & & & \\
 & & h_{m+i,0} & \xrightarrow{P^1} & h_{m+i-1,1} & \xrightarrow{P^p} & h_{m+i-2,2} & \xrightarrow{P^{p^2}} & \cdots
\end{array}$$

$$h_{m+i,m} \xrightarrow{\beta} b_{m+i,m-1} \xrightarrow{P^1} b_{m+i,m-2}^{p} \xrightarrow{P^p} b_{m+i,m-3}^{p^2} \xrightarrow{P^{p^2}} \cdots$$

Elements shown above that are linked by these operations all have the same Hill-Lawson filtration.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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Elements shown above that are linked by these operations all have the same Hill-Lawson filtration. This is not true for the Adams and Novikov filtrations.

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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The action of internal Steenrod operations in the Hill-Lawson E_1-term for each $i > 0$ is shown below.

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h_{m+i,0} & \xrightarrow{P^1} & h_{m+i-1,1} & \xrightarrow{P^p} & h_{m+i-2,2} & \xrightarrow{P^{p^2}} & \cdots
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Elements shown above that are linked by these operations all have the same Hill-Lawson filtration. This is not true for the Adams and Novikov filtrations. Each sequence has finite length because one of the subscripts in it eventually gets too small.

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

Some Hill-Lawson d_1s

It is easy to show that $d_1 v_{2m+i} = v_m h_{m+i,m}$.

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1s
A possible $\mathbb{E}_2$ structure
Conclusion

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It is easy to show that $d_1 v_{2m+i} = v_m h_{m+i,m}$. Differentials must commute with internal Steenrod operations, so for each $i >$ we get a diagram

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1s
A possible $\mathbb{E}_2$ structure
Conclusion

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{\scriptstyle d_1}\uparrow & & {\scriptstyle d_1}\uparrow & & {\scriptstyle d_1}\uparrow & & \\
v_{2m+i} & \xrightarrow{\beta} & h_{2m+i,0} & \xrightarrow{P^1} & h_{2m+i-1,1} & \xrightarrow{P^p} & \cdots
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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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These d_1s correspond to the d_{p^j}s that Mahowald, Shick and I wanted in the localized Adams spectral sequence!

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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This is why I like the Hill-Lawson spectral sequence.

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

A possible $\mathbb{E}_2$ structure

Since $y(m)$ is the Thom spectrum associated with a loop map (but not a double loop map),

Doug Ravenel

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

A possible $\mathbb{E}_2$ structure

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The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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It is known that any $\mathbb{E}_1$ ring spectrum R has an $\mathbb{E}_2$ center $\mathfrak{z}(R)$, AKA its topological Hochschild cohomology.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

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$$\mathfrak{Z}(y(m)) \simeq F(J_{p^m-1}S^2, y(m)),$$

a certain function spectrum.

The Hill-Lawson spectral sequence and the telescope conjecture

Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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Doug Ravenel

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

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$$\mathfrak{Z}(y(m)) \simeq F(J_{p^m-1}S^2, y(m)),$$

a certain function spectrum. Its Hill-Lawson filtration may or may not be compatible with its $\mathbb{E}_2$ structure. If it is, the spectral sequence has certain Dyer-Lashof operations.

The Hill-Lawson spectral sequence

The MRS spectrum $y(m)$

Inverting v_m

The Adams-Novikov spectral sequence

The localized Adams spectral sequence

The localized Hill-Lawson spectral sequence for $y(m)$

Internal Steenrod operations for $y(m)$

Some Hill-Lawson d_1 s

A possible $\mathbb{E}_2$ structure

Conclusion

A possible $\mathbb{E}_2$ structure (continued)

If we have the desired $\mathbb{E}_2$ structure, we get the following diagram for each $i > 0$, where the horizontal arrows are Dyer-Lashof operations.

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
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A possible $\mathbb{E}_2$ structure (continued)

If we have the desired $\mathbb{E}_2$ structure, we get the following diagram for each $i > 0$, where the horizontal arrows are Dyer-Lashof operations.

$$\begin{array}{ccccccc}
v_m h_{m+i,m} & \xrightarrow{Q_1} & v_m^p h_{m+i,m+1} & \xrightarrow{Q_1} & v_m^{p^2} h_{m+i,m+2} & \xrightarrow{Q_1} & \cdots \\
\uparrow{\scriptstyle d_1} & & \uparrow{\scriptstyle d_p} & & \uparrow{\scriptstyle d_{p^2}} & & \\
v_{2m+i} & \xrightarrow{Q_0} & v_{2m+i}^p & \xrightarrow{Q_0} & v_{2m+i}^{p^2} & \xrightarrow{Q_0} & \cdots
\end{array}$$

The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
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These longer Hill-Lawson differentials correspond to d_1s in both the Adams and Adams-Novikov spectral sequences.

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
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I will call them Dyer-Lashoff differentials.

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The Hill-Lawson spectral sequence
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Inverting v_m
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Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1s
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Conclusion

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After taking these Steenrod and Dyer-Lashoff differentials into account, we would be left with

$$R_m \otimes E(h_{m+i,j} : i + j \leq m) \otimes P(b_{m+i,j})/(b_{m+i,j}^{p_{m-1-j}}).$$

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
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The localized Hill-Lawson spectral sequence for $y(m)$
Internal Steenrod operations for $y(m)$
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The MRS spectrum $y(m)$

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Internal Steenrod operations for $y(m)$

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
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Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion

The Hill-Lawson spectral sequence

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Inverting v_m

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THANK YOU!

Table of spectral sequence filtrations and dimensions

Filtrations			
Spectral sequence	v_{m+n}	$h_{m+i,j}$	$b_{m+i,j}$
Adams-Novikov	0	1	2
Adams	1	1	2
Hill-Lawson	p^n	p^{i+j}	p^{i+j+1}

Element	Dimension
v_{m+n}	$2p^{m+n}-2$
$h_{m+i,j}$	$2p^j(p^{m+i}-1)-1$
$b_{m+i,j}$	$2p^{j+1}(p^{m+i}-1)-2$

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The Hill-Lawson spectral sequence
The MRS spectrum $y(m)$
Inverting v_m
The Adams-Novikov spectral sequence
The localized Adams spectral sequence
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Internal Steenrod operations for $y(m)$
Some Hill-Lawson d_1 s
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Hill-Lawson differentials

Steenrod differentials:

$$\begin{array}{ccccccc}
v_m h_{m+i,m} & \xrightarrow{\beta} & v_m b_{m+i,m-1} & \xrightarrow{P^1} & v_m b^p_{m+i,m-2} & \xrightarrow{P^p} & \cdots \\
d_1 \uparrow & & d_1 \uparrow & & d_1 \uparrow & & \\
v_{2m+i} & \xrightarrow{\beta} & h_{2m+i,0} & \xrightarrow{P^1} & h_{2m+i-1,1} & \xrightarrow{P^p} & \cdots
\end{array}$$

Dyer-Lashof differentials:

$$\begin{array}{ccccccc}
v_m h_{m+i,m} & \xrightarrow{Q_1} & v_m^p h_{m+i,m+1} & \xrightarrow{Q_1} & v_m^{p^2} h_{m+i,m+2} & \xrightarrow{Q_1} & \cdots \\
d_1 \uparrow & & d_p \uparrow & & d_{p^2} \uparrow & & \\
v_{2m+i} & \xrightarrow{Q_0} & v^p_{2m+i} & \xrightarrow{Q_0} & v^{p^2}_{2m+i} & \xrightarrow{Q_0} & \cdots
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Some Hill-Lawson d_1 s
A possible $\mathbb{E}_2$ structure
Conclusion