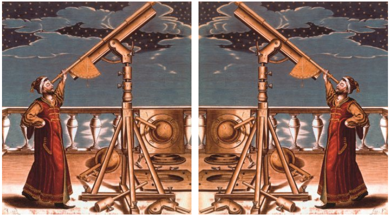
Two equivariant approaches to the telescope conjecture

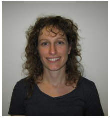
MIT Topology Seminar

November 20, 2017



Doug Ravenel
University of Rochester

This talk began in discussions last year with



Agnes Beaudry

Mark Behrens

Prasit Bhattacharya

Dominic Culver

Zhouli Xu

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Doug Ravenel

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LOCALIZATION WITH RESPECT TO CERTAIN PERIODIC HOMOLOGY THEORIES

By Douglas C. Ravenel*

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Let X be a p-local finite spectrum with $K(n)_*X \neq 0$ and $K(n-1)_*X = 0$. Such complexes are know to exist for all n and p by a theorem of Steve Mitchell.

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The $n = 1$ case was proved by Mahowald for $p = 2$ and by Miller for odd primes in 1981.

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Earthquake of October 17, 1989

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I will outline the construction of $y(n)$ later in the talk.

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I will describe two different ways we might do this. It is too early to tell if either approach will work.

The construction of $y(n)$

Consider the diagram

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We know that

$$H_*\Omega^2 S^3 = \mathbf{Z}/2[u_1, u_2, \ldots] \qquad \text{with } |u_n| = 2^n - 1 = |\xi_n|.$$

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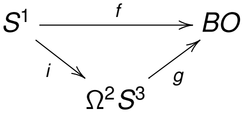

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We will construct subspaces Y_n of $\Omega^2 S^3$ with

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He showed there is a 2-local fiber sequence

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Our space Y_n is $\Omega J_{2^n-1}S^2$, so it maps to $\Omega^2 S^3$ as desired. The MRS spectrum $y(n)$ is the Thomification of

$$\Omega J_{2^n-1}S^2 \longrightarrow \Omega^2 S^3 \xrightarrow{\ g\ } BO.$$

The construction of $y(n)$ (continued)

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we get maps of spectra

$$\Sigma^\infty S^{|v_n|} \to \Sigma^\infty \Omega^3 S^{2^{n+1}+1} \to y(n) \to H\mathbf{Z}/2.$$

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Going equivariant I

If the MRS approach is to succeed, we need some more structure in the localized Adams spectral sequence for $Y(n)$.

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Going equivariant I

If the MRS approach is to succeed, we need some more structure in the localized Adams spectral sequence for $Y(n)$. Here I will outline the first of two ways to get $y(n)$ and $Y(n)$ into a C_2-equivariant setting. Each of them will be a retract of the fixed point set of a C_2-spectrum.

Going equivariant I

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Recall that the construction of $y(n)$ involved the diagram

$$\begin{array}{ccccc} S^1 & \xrightarrow{\ i\ } & \Omega^2 S^3 & \xrightarrow{\ g\ } & BO \\ & & \uparrow & & \\ & & \Omega J_{2^n-1} S^2 & & \end{array}$$

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We can add another space and get

$$\begin{array}{ccccc} S^1 & \xrightarrow{i} & \Omega^2 S^3 & \xrightarrow{g} & BO \\ & & \uparrow & & \uparrow \\ & & \Omega J_{2^n-1} S^2 & \to & \Omega(SU(k+1)/SO(k+1)) \end{array} \quad \text{for } k \gg 0.$$

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Going equivariant (continued)

$$\begin{array}{ccccc} S^1 & \xrightarrow{i} & \Omega^2 S^3 & \xrightarrow{\quad g \quad} & BO \\ & & \uparrow & & \uparrow{\scriptstyle a_k} \\ & & \Omega J_{2^n-1} S^2 & \xrightarrow{g_n} & \Omega(SU(k+1)/SO(k+1)). \end{array}$$

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$$H_* BO = \mathbf{Z}/2[b_1, b_2, \ldots] \quad \text{where } |b_i| = i,$$
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Going equivariant (continued)

One can show that

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is the fixed point set of the following diagram of C_2-spaces:

$$\begin{array}{ccccc} S^\rho & \xrightarrow{i} & \Omega^{1+\rho} S^{1+2\rho} & \xrightarrow{g} & BU_{\mathbf{R}} \\ & & \uparrow & & \uparrow a_k \\ & & \Omega^\rho J_{2^n-1} S^{2\rho} & \xrightarrow{g_n} & \Omega^\sigma SU(k+1)_{\mathbf{R}} \end{array}$$

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Going equivariant (continued)

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where

- $BU_{\mathbf{R}}$ and $SU_{\mathbf{R}}$ denote the spaces BU and SU equipped with a C_2-action related to complex conjugation,

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Going equivariant (continued)

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- $BU_{\mathbf{R}}$ and $SU_{\mathbf{R}}$ denote the spaces BU and SU equipped with a C_2-action related to complex conjugation,
- σ denotes the sign representation of C_2 and
- $\rho = 1 + \sigma$ denotes its regular representation.

Going equivariant (continued)

Here is our C_2-diagram again.

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Going equivariant (continued)

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$$\begin{array}{ccccc}
S^\rho \xrightarrow{\ i\ } \Omega^{1+\rho} S^{1+2\rho} & \xrightarrow{\quad g \quad} & BU_{\mathbf{R}} & & MU_{\mathbf{R}} \\
\uparrow & & \uparrow i_k & & \uparrow \\
\Omega^\rho J_{2^n-1} S^{2\rho} & \xrightarrow{\ g_n\ } & \Omega^\sigma SU(k+1)_{\mathbf{R}} & & X(k)_{\mathbf{R}}
\end{array}$$

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with Thom spectra indicated on the right.

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with Thom spectra indicated on the right. Taking 2-local fibers of the vertical maps in the square gives

$$\begin{array}{ccc}
\Omega^{1+\rho}S^{1+2\rho} & \xrightarrow{g} & BU_{\mathbf{R}} \\
\uparrow & & \uparrow{\scriptstyle a_k} \\
\Omega^\rho J_{2^n-1}S^{2\rho} & \xrightarrow{g_n} & \Omega^\sigma SU(k+1)_{\mathbf{R}} \\
\uparrow & & \uparrow \\
\Omega^{2+\rho}S^{1+2^{n+1}\rho} & \longrightarrow & \Omega^\rho(SU/SU(k+1))_{\mathbf{R}}
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The two fibers have the same connectivity when
$k = 2^{n+1} - 2 = |v_n|$.

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Going equivariant (continued)

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\Omega^{1+\rho} S^{1+2\rho} & \xrightarrow{\quad g \quad} & BU_{\mathbf{R}} \\
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It follows that we have a map $y(n) \to w(|v_n|)$ inducing a monomorphism in mod 2 homology,

Going equivariant (continued)

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$$S^{|v_n|} \to \Omega^3 S^{2^{n+1}+1} \to y(n) \to w(|v_n|),$$

Going equivariant (continued)

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\Omega^{1+\rho}S^{1+2\rho} & \xrightarrow{\quad g \quad} & BU_{\mathbf{R}} \\
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where $w(k)$ is the geometric fixed point set of the Thom spectrum $X(k)_{\mathbf{R}}$.

Going equivariant (continued)

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Going equivariant (continued)

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where $w(k)$ is the geometric fixed point set of the Thom spectrum $X(k)_{\mathbf{R}}$. The above composite leads to a telescope $W(|v_n|)$ which is the geometric fixed point spectrum of the telescope for a map

$$\Sigma^{(1+|v_n|)\rho-1} X(|v_n|)_{\mathbf{R}} \to X(|v_n|)_{\mathbf{R}}.$$

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Going equivariant (continued)

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\Omega^{1+\rho}S^{1+2\rho} & \xrightarrow{\quad g \quad} & BU_{\mathbf{R}} \\
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The underlying spectrum of this telescope is contractible

Going equivariant (continued)

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$$\Sigma^{(1+|v_n|)\rho-1} X(|v_n|)_{\mathbf{R}} \to X(|v_n|)_{\mathbf{R}}.$$

The underlying spectrum of this telescope is contractible because the underlying map is known to be nilpotent.

Going equivariant II

In this approach,

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Going equivariant II

In this approach, $y(n)$ and $Y(n)$ will be given nontrivial C_2-actions.

Going equivariant II

In this approach, $y(n)$ and $Y(n)$ will be given nontrivial C_2-actions. They will be the underlying spectra of a pair of C_2-spectra.

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Going equivariant II

In this approach, $y(n)$ and $Y(n)$ will be given nontrivial C_2-actions. They will be the underlying spectra of a pair of C_2-spectra.

Recall that the starting point of the construction of $y(n)$ was the diagram

$$\begin{array}{ccc} S^1 & \xrightarrow{\quad f \quad} & BO \\ & {}_{i}\searrow \qquad \nearrow_{g} & \\ & \Omega^2 S^3. & \end{array}$$

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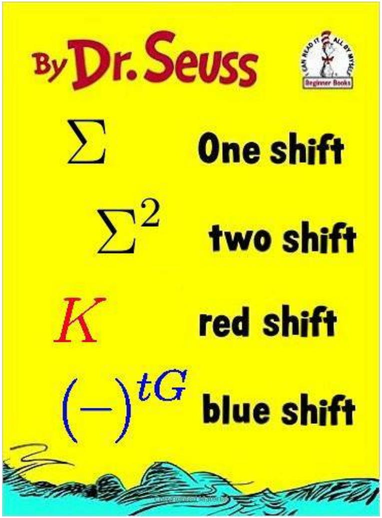

THANK YOU!