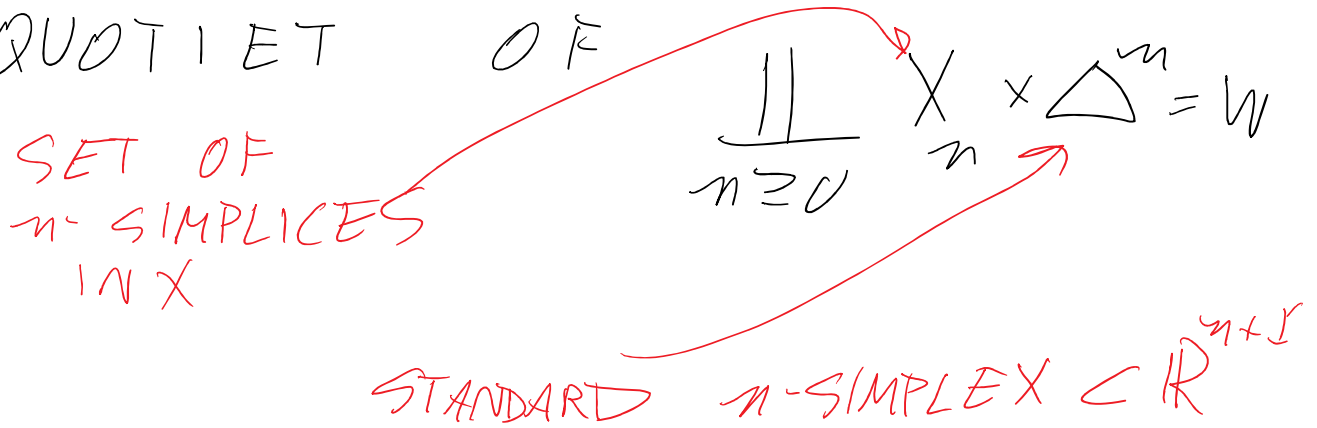
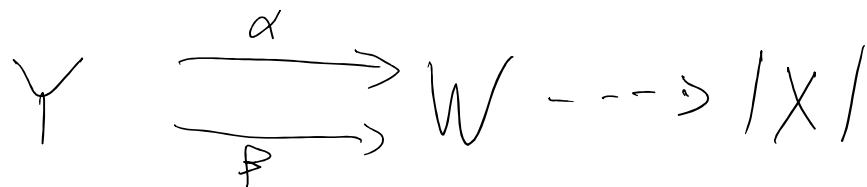


FOR A SIMPLICIAL SET  $X$ ,  
THERE IS A TOP. SPACE  $|X|$ ,  
ITS GEOMETRIC REALIZATION

WHICH IS A CERTAIN TOPOLOGICAL  
QUOTIENT OF



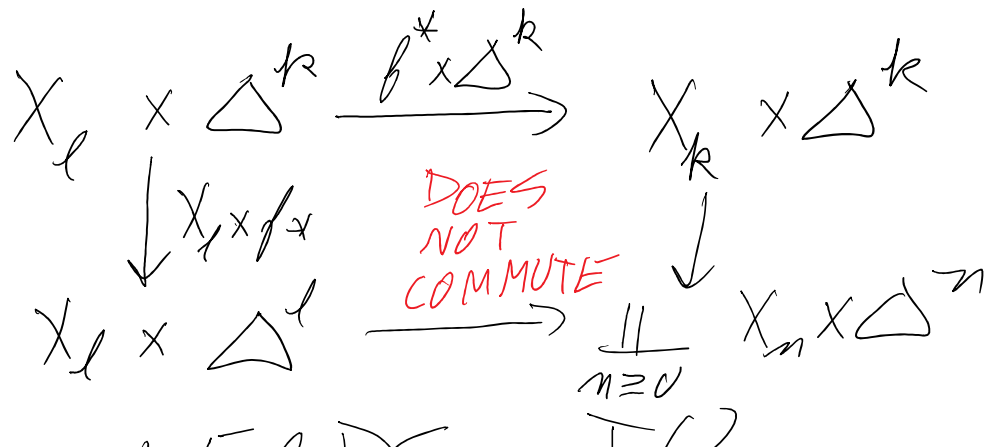
THIS WILL BE A CO-EQUALIZER  
FOR



TO DEFINE  $Y$ :

FOR MORPHISM  $[k] \rightarrow [l]$  IN  $\Delta$

WE GET A DIAGRAM



THIS

THIS LEADS TO

$$W'_0 = \coprod_{b: [k] \rightarrow [l]} X_k \times \Delta^k \begin{array}{c} \xrightarrow{\text{HORIZONTAL}} \\ \xrightarrow{\text{VERTICAL}} \end{array} \coprod_{n \geq 0} X_n \times \Delta^n \begin{array}{c} \parallel \\ W \end{array}$$

SUFFICES TO CONSIDER ONLY THE FACE + DEGENERACY MAPS IN  $\Delta$ .

THE COEQUALIZER OF H AND V IS  $|X|$  BY DEFINITION.

EXAMPLE  $|\Delta[n]|\ = \Delta^n$

THIS DEFINES A FUNCTOR

$$\text{Set}_{\Delta} \xrightarrow{|\cdot|} \text{Spaces}$$

CATEGORY OF SIMPLICIAL SETS

WHAT IS  $\text{Set}_{\Delta}$ ? IT IS THE CATEGORY WHOSE

OBJECTS ARE SIMPLICIAL SETS

A MORPHISM  $X \xrightarrow{f} Y$  OF SIMPLICIAL SETS  $\Leftrightarrow$  A COLLECTION OF MAPS

$$\begin{array}{ccc}
 X_n & \xrightarrow{f_n} & Y_n \\
 d_i \downarrow & & \downarrow d_i \\
 X_{n-1} & \xrightarrow{f_{n-1}} & Y_{n-1} \\
 s_j \downarrow & & \downarrow s_j \\
 X_n & \xrightarrow{f_n} & Y_n
 \end{array}$$

THERE IS ANOTHER FUNCTOR

$$\text{Spaces} \xrightarrow{\text{Sing}} \text{Set}_{\Delta}$$

LET  $X$  BE A SPACE, LET

$\text{Sing}(X)_n =$  THE SET OF ALL CONTINUOUS MAPS

$$\Delta^n \xrightarrow{\sigma} X$$

$$\text{Set}_n \xrightarrow{\text{Sing}} \text{Spaces}$$

Set  $\triangleleft \xleftarrow[\text{Sing}]{L}$  Spaces

THESE ARE ADJOINT  
TO EACH OTHER  
FOR A SIMP. SET  $X$   
AND TOP. SPACE  $Y$

Set  $\triangleleft (X, \text{Sing}(Y))$

AND Map  $(|X|, Y)$

ARE NATURALLY ISOMORPHIC.  
THEY INDUCE AN  
EQUIVALENCE OF  
CATEGORIES.

THERE IS A FUNCTOR

$C: \text{Set} \triangleleft \longrightarrow CC =$  CATEGORY  
OF CHAIN  
COMPLEXES

LET  $X$  BE A SIMP. SET

$C_n(X)$  = FREE ABELIAN GP ON THE SET  $X_n$

WE NEED A BOUNDARY OPERATOR

$$C_n(X) \longrightarrow C_{n-1}(X)$$

IN  $\Delta$  WE HAVE FACE MAPS

$$[n-1] \xrightarrow{d_i} [n] \quad 0 \leq i \leq n$$

$$\{0, 1, \dots, n-1\} \rightarrow \{0, 1, \dots, n\}$$

$[n] \ni i$  IS NOT IN THE IMAGE

$$X_{n-1} \xleftarrow{d_i} X_n$$

$$C(X_{n-1}) \xleftarrow{d_i} C(X_n) \quad 0 \leq i \leq n$$

$$C_{n-2}(X) \xleftarrow{\sum_{0 \leq j \leq n-1} (-1)^j d_j} C_{n-1}(X) \xleftarrow{\sum_{0 \leq i \leq n} (-1)^i d_i} C_n(X)$$

THE SIMPLICIAL IDENTITIES IMPLY THAT THIS COMPOSITE HOMOMORPHISM IS ZERO.

GIVEN A <sup>TOP</sup> SPACE  $X$ , ...

WE HAVE A SIMPLICIAL  $\text{Sing}(X)$   
 AND A CHAIN COMPLEX  
 $C(\text{Sing}(X)) =: S(X)$ ,  
 THE SINGULAR CHAIN COMPLEX

OF  $X$ .

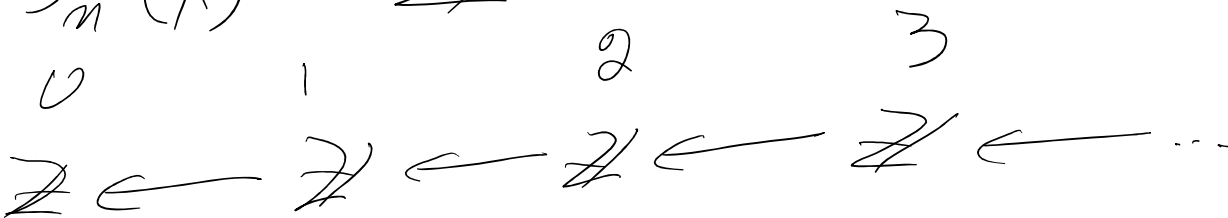
WE DEFINE  $H_n(X) = H_n(S(X))$

HOW DO WE DEAL  
 WITH THIS ???

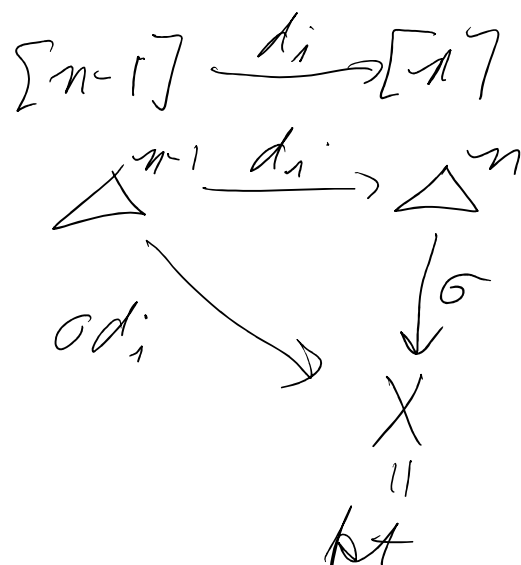
EXAMPLE :

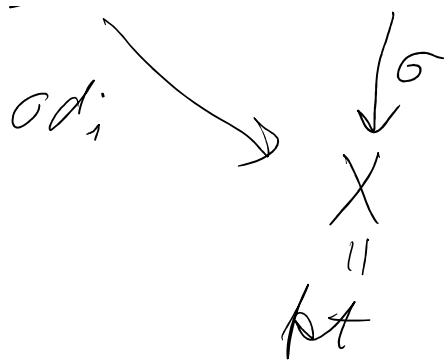
$X = \text{pt} = \text{SINGLE POINT}$

$$S_n(X) = \mathbb{Z}$$



A FACE MAP  
 INDUCES





THIS INDUCES

$$S_n(\text{pt}) \longrightarrow S_{n-1}(\text{pt})$$

$$\parallel \quad \quad \quad \parallel$$

$$\mathbb{Z} \xrightarrow{1} \mathbb{Z}$$

$$0 \leftarrow \mathbb{Z} \xleftarrow{0} \mathbb{Z} \xleftarrow{1} \mathbb{Z} \xleftarrow{2} \mathbb{Z} \xleftarrow{\dots} \mathbb{Z} \xleftarrow{n} \mathbb{Z}$$

$$\sum_{0 \leq i \leq n} (-1)^i = \begin{cases} 0 & \text{IF } n \text{ IS ODD} \\ 1 & \text{IF } n \text{ IS EVEN} \end{cases}$$

$$H_0 = \mathbb{Z} \quad H_1 = 0 \quad H_2 = 0 \quad \dots$$

CONCLUSION

$$H_n(\text{pt}) = \begin{cases} \mathbb{Z} & \text{FOR } n=0 \\ 0 & \text{FOR } n>0 \end{cases}$$

THIS IS THE DIMENSION AXIOM

WE ALSO NEED TO DEFINE

$H_*(X, A)$ , FOR  $A \subset X$ , CALLED  
RELATIVE HOMOLOGY.

$$0 \rightarrow S(A) \xrightarrow{i-1} S(X) \rightarrow S(X)/S(A) \rightarrow 0$$

LET  $H_*(X, A) := H(S(X)/S(A))$   
THIS LEADS TO A LONG  
EXACT SEQUENCE

$$\dots \rightarrow H_n(A) \rightarrow H_n(X) \rightarrow H_n(X, A) \rightarrow H_{n-1}(A) \rightarrow \dots$$

THIS THE EXACTNESS  
AXIOM.