GARAGE CODE * 5111 <enter>

Foling Zou — The odd primary Arf invariant thm

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The methods here are used in the detection thm.

Recall the Adams SS $E_2^{s,t} = \mathrm{Ext}_{A_p}^{s,t}(\mathbb{F}_p, \mathbb{F}_p)$

$$\Rightarrow \pi_{t-s}(S^0) \otimes \mathbb{Z}_p$$

$$d_r : E_r^{s,t} \longrightarrow E_r^{s+r, t+r-1}$$

$p=2$ $h_i \in \mathrm{Ext}^{1,2^i} \longleftrightarrow Sq^{2^i} \in \mathcal{A}_2$

Thm (Adams) For $i \geq 4$, $d_2(h_i) = h_0 h_{i-1}^2$ and h_i survives for $0 \leq i \leq 3$

Related to Hopf invariant 1.

$p>2$ $h_i \in \mathrm{Ext}^{1, q p^i} \longleftrightarrow P^{p^i}$, $q = 2p-2$

Thm (Liulevicius) For $i \geq 1$, $d_2(h_i) = a_0 b_{i-1}$ where and h_0 survives, b_{i-1} is analogous to h_{i-1}^2

The Arf invar elts for $p=2$ are h_j^2 related to θ_j by Browder's thm.

For $p>2$, $b_i = \langle h_i, \ldots h_i \rangle$ = p-fold Massey product

[aside $H^*(C_p; \mathbb{Z}/p) = \begin{cases} \mathbb{Z}/2[h] & p=2 \\ E(h) \otimes P(b) & b \in H^2 \end{cases}$, $b = \langle h, \ldots h \rangle$]

The b_i corresponds to Adem relation for $P^{(p-1)p^i} P^{p^i}$

For $p>3$ we show $d_{2p-1}(b_i) \neq$ for $i \geq 1$

Not sure about $p=3$

Thm (Toda) $d_{2p-1}(b_1) = h_0 b_0^p$

May showed $h_0 b_1^3 = 0$ in E_7 for $p=3$

We can hope for $d_{2p-1}(b_{i+1}) = h_0 b_i^p$

We will use ANSS instead

Recall $BP_* = \mathbb{Z}_{(p)}[v_1, v_2 \cdots]$ $\quad |v_i| = |t_i| = 2(p^i - 1)$

$BP_*(BP) = BP_*[t_1, t_2, \cdots]$

$$E_2^{s,t} = \mathrm{Ext}^{s,t}_{BP_*(BP)}(BP_*, BP_*) \Rightarrow \pi_{t-s}(S^0) \otimes \mathbb{Z}_{(p)}$$

The map $BP \longrightarrow H\mathbb{Z}/p \rightsquigarrow \text{ANSS} \longrightarrow \text{ASS}$

Thm For $p>3$, $i>0$ then b_i does not survive

Step 1: Find preimage of b_i in ANSS, namely

$$\sum_{0<j<p^{i+1}} p^{-1}\binom{p^{i+1}}{j}[t_i^j | t_i^{p^{i+1}-j}]$$

Thm For $p \geq 3$, $d_{2p-1}(b_{i+1}) \neq 0$ in ANSS

Step 1.1 Show $d_{2p-1}(b_{i+1}) = h_0 b_i^p$ mod ker $b_0^{a_i}$

Use induction on Toda's result and relation

$h_{i+1} b_i^p = h_{i+2} b_0^p$ $\qquad a_i = \frac{p(p^i-1)}{p-1}$

Step 1.2 Show target nontrivial.

$b^I, h_0 b^I \neq 0$ for any monomial b^I

Idea Use Morava stabilizer gp

$$\mathrm{Ext}_{BP_* BP}(BP_*, BP_*) \longrightarrow \mathrm{Ext}_{\mathbb{F}_p[S_n]}(\mathbb{F}_p, \mathbb{F}_p) \to \mathrm{Ext}_{\mathbb{F}_p[C_p]}(\mathbb{F}_p, \mathbb{F}_p)$$

known as above

$h_0 \longmapsto$ scalar $\cdot h$

$b_i \longmapsto$ scalar b

scalars $\neq 0$

STEP 2 Relate to Adam SS for $p \geq 5$.

Assume $x = b_{i+1}$ survives, so $\tilde{x} \mapsto x$ survives

[illegible]

Assume $x = b_{i+1}$ survives, so $\tilde{x} \mapsto x$ survives in ANSS, but $\hat{x}$ cannot exist by low filtration

There is nothing in filtration < 2

If $\hat{x} \in Ext^2$, $\hat{x} = b_{i+1} + y$. The gp Ext^2 is known by MRW. It is generated by

$$\beta_{a_{i,j}/p^{i+3-2j}}, \quad j = 1, 2, \dots \left\lfloor \frac{i+3}{2} \right\rfloor$$

$j = 1$ $\beta_{p^{i+1}/p^{i+1}}$ $\quad j > 1$ noise

The elts for $j > 1$ are in kernel of Thom reduction.

Lemma $Ext_{BP_*BP}(BP_*, BP_*) \to Ext_{BP_*BP}(BP_*/BP_*/I_3)$

noise $\longmapsto 0$

Thm (L. Smith) $\exists\ S^0 \xrightarrow{f} V(2)$ realizing $BP_* \to BP_*/I_3$.
for $p \geq 5$

Candidate for $d_{2p+1}(b_{i+1} + y)$ $\qquad n = p-1$

$\Gamma = BP_*(BP)$

$$\begin{tikzcd}
Ext_\Gamma(BP_*, BP_*) \arrow[rr] \arrow[dr, "f_*"'] & & Ext_\Gamma(BP_*, v_n^{-1} BP_*/I_n) \\
& Ext_\Gamma(BP_*, BP_*/I_3) \arrow[ur] &
\end{tikzcd}$$

$f_*(y) = 0$

DEAD BATTERY