A RELATION BETWEEN TWO MODULI SPACES STUDIED BY V.G. DRINFELD

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1. Introduction

The local Jacquet-Langlands correspondence gives a bijection between supercuspidal representations of the general linear group $GL(h)$ of a local field K and irreducible representations of the multiplicative group of the central division algebra of invariant $1/h$. By the local Langlands conjecture these should also be in bijection with the irreducible h-dimensional representations of the Weil-group W_K. H. Carayol conjectured two ways to realize these correspondences, namely via the cohomology of either coverings of Drinfeld's symmetric space, or of the universal deformation of the one dimensional formal group of height h (see [3]). A result along these lines has been shown by Harris and Taylor ([13]), while the author has proven by much more elementary means that the first model incorporates at least the Jacquet-Langlands correspondence ([7]). The purpose of the present paper is to relate the two models via a geometric construction. Namely the corresponding maximal covers (infinite level-structures) become isomorphic as rigid spaces.

A rough idea of what is going one can be given by the associated period-maps. These map the Tate-module into the tensorproduct of crystalline cohomology and Fontaine's ring of periods A_{crys}. In our case the Tate-module is always trivialised (infinite level-structure), and the crystalline cohomology is (up to torsion) constant. For the one-dimensional formal $\mathcal{O}_K$-modules H the period map sends $T_\pi(H) = \mathcal{O}_K^h$ to its crystalline homology W which admits an action by the integers $\mathcal{O}_B$ of the division-algebra of invariant $1/h$. For the Drinfeld symmetric space one classifies formal $\mathcal{O}_B$-modules of dimension h, and the period-map sends $\mathcal{O}_B$ into W^h and is $\mathcal{O}_B$-linear. Obviously these types of maps correspond one to one, where the Tate-module $T_\pi(H)$ changes sides from étale to crystalline and is replaced by its dual.

One defect of the present paper is that we reduce formal $\mathcal{O}_K$-modules to formal groups. It would be more satisfactory to develop an analogue of the classical theory of finite flat group-schemes and p-divisible groups for $\mathcal{O}_K$-modules. For example the multiplicative group should be replaced by a Lubin-Tate formal group, Cartier-duality by maps into Lubin-Tate, divided powers by π-divided powers ([11]), A_{crys} by an analogue which is naturally an algebra over $\mathcal{O}_K$, etc.. This also should take care of positive characteristics. Unfortunately such a theory does not exist, or at least is not known to me, and everything is reduced prosaically to $\mathbb{Q}_p$. (In the meanwhile such a theory has been constructed.)

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gives me special pleasure to dedicate this work to A.N.Parshin because I profited enormously from his work on diophantine geometry.

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2. Preliminaries

Throughout the paper K denotes a finite extension of the p-adic numbers $\mathbb{Q}_p$, that is a non archimedean local field of characteristic zero. Also B denotes the division-algebra of invariant $1/h$ over K, h an integer. The integers in K or B are denoted by $\mathcal{O}_K$, respectively $\mathcal{O}_B$. A formal group G of dimension d over a p-adically complete (commutative) ring A is given by a formal group law on the powerseries $A[[T_1,...T_d]]$. It is of finite height h if multiplication by p induces a finite map of degree p^h on $A[[T_1,...T_d]]$. In this case there exists a universal vectorextension

$$0 \to F \to EG \to G \to 0,$$

with F an additive formal group of dimension $h-d$. Furthermore EG is crystalline, that is if $I \subset A$ is a PD-ideal then up to canonical isomorphism EG depends only on the reduction of G modulo I. Also maps (of formal groups) defined modulo I extend canonically to universal vectorextensions (see [15]). For example we can apply this to $I = p \cdot A$ with its canonical PD-structure. Thus if A admits a Frobenius-lift the Frobenius on G modulo p extends to a semilinear endomorphism of EG. Also all this extends to p-divisible groups. We call the Lie-algebra of EG $M_1(G)$ (crystalline homology).

If V is a complete discrete valuation ring of mixed characteristic, with perfect residue field k, we can form the ring A_{crys} (see [10]). It has a PD-ideal, a Frobenius-lift, and an action of $Gal(\bar{V}/V)$. If G denotes a p-divisible group over V its universal vectorextension depends up to isogeny only on the fibre of G over k. Thus if G_0 denotes any lift of $G \otimes_V k$ over the Witt-vectors $V_0 = W(k)$, then $M_1(G)[1/p] = M_1(G_0) \otimes_{V_0} V[1/p]$, and this admits a semilinear Frobenius automorphism (the inverse of the map induced by Frobenius on EG_0). Furthermore if $T_p(G)$ denotes the Tate-module of G any $\rho \in T_p(G)$ defines over $\bar{V}$ a homomorphism from $\mathbb{Q}_p/\mathbb{Z}_p$ into G, which lifts over $A_{crys}(V)$ to a map of universal vectorextensions, thus to a map from $\mathbb{Q}_p$ into $EG(A_{crys}(V))$. Also the induced map on Lie-algebras defines an element of the Lie-algebra of EG, so finally a canonical period-map (as in [8])

$$T_p(G) \to M_1(G_0) \otimes A_{crys}(V)[1/p].$$

This period map respects the Galois-action and the image is invariant under Frobenius and lies in the first stage of the product filtration. If we apply this to the multiplicative group (with the canonical generator of $M_1(\mathbb{G}_m)$) we obtain as image of a generator of $\mathbb{Z}_p(1)$ a canonical element $t \in F^1(A_{crys}(V))$, well defined up to multiplication by the units $\mathbb{Z}_p^*$. For general G we consider the determinant of the period-map. For this we choose generators of the determinants of $T_p(G)$ and of $M_1(G_0)$, which are well-determined up to units in $\mathbb{Z}_p^*$, respectively V_0^*. Then the determinant lies in $F^d(A_{crys}(V)$.

Lemma 1. *The determinant lies in $V_0^* \cdot t^d$.*

Proof: We may assume that k is algebraically closed. As the Galois-action on the determinant of $T_p(G)$ is via the d-th power of the cyclotomic character (see [16]) the determinant must be a K_0-multiple of t^d (see [10]). We have to show that the factor is a unit. Also choosing a generator of the determinant of $M_1(G_0)$ which is a Frobenius-eigenvector with eigenvalue p^{-d} we get that the factor is Frobenius-invariant, that is it lies in $\mathbb{Q}_p$. (We could have derived this directly from [10] without using the Galois-action, but this result is more delicate).

This assertion is compatible with isogenies and extensions, and also with Cartier-duality (the composition of the period map for G with the adjoint of that for G^{dual} is t^h). This already implies that the factor is nonzero. Also by a deformation argument we may assume that $G = G_0$: There exists a formal group $\mathcal{G}$ over $V[[T]]$ with fibre G_0 at $T = 0$ and fibre G at $T = \pi$, for a uniformiser π of V. Adjoining the squareroot of π and scaling T by it we may assume that $\mathcal{G}$ is constant modulo π. Then the theory of A_{crys} works over $V[[T]]$ (see [8], although we only need a rather elementary case of the general theory essentially covered in [6]). Thus a period of $\mathcal{G}$ in $F^d(A_{crys}(V[[T]]))$. Using the Galois-action we see that modulo F^{d+1} it is a multiple of t^d, with the factor in $V[[T]][1/p]$. Mapping to the localisation of $V[[T]]$ in $\pi \cdot V[[T]]$ gives that the factor lies in $\mathbb{Q}_p$, and by pushout coincides with that for G_0, respectively G.

So it suffices to consider G_0, which can be an arbitrary lift of its special fibre. Using extensions and isogenies we may assume that G_0 is a formal group of dimension d and height h, with d and h coprime and $0 < d < h$. We also may assume that G_0 admits as endomorphisms the ring R of integers in the unramified extension $\mathbb{Q}_{p^h}$ of degree h of $\mathbb{Q}_p$. It then follows that $T_p(G)$ is a free R-module of rank one, and that $Gal(\bar{V}/V)$ acts on it via a character $\chi_G : Gal(\bar{V}/V) \to R^*$. It is known that χ_G is the product of d conjugates of the Lubin-Tate character $\chi : Gal(\bar{V}/V) \to R^*$. The conjugations are via i-th powers of Frobenius, where $0 \leq i < d$ ranges over those numbers for which the corresponding eigenspace of R on $Lie(G_0)$ does not vanish.

Furthermore $T_p(G_0) \otimes R$ is the direct sum of eigenspaces under R (acting via endomorphims of G_0). We index them by integers i, $0 \leq i < h$, such that R acts on i-space via the i-th power of Frobenius. Similarly we have such a decomposition for $M_1(G_0)$. Now the period-map is integral (no need to invert p), and as it respects endomorphisms it acts (after choice of generators) on the i-space by an element $x_i \in A_{crys}(V)$. This element lies in $F^1(A_{crys}(V))$ if the corresponding basis-element of $M_1(G_0)$ does not lie the Lie-algebra of the vectorpart F of EG_0, which happens for d indices i. Furthermore the x_i are eigenvectors for $Gal(\bar{V}/V)$, with character the i-th Frobenius conjugate of χ_G. Finally x_i (and thus also y_i) divides t because the period-map has an inverse up to t. The leading term of x_i in $gr_j^F(A_{crys}(V)) \cong \hat{\bar{V}}$ ($j = 0, 1$) has a certain valuation $v_i \in \mathbb{Q}$ (valuations are normalised such that p has valuation 1). Modulo elements of valuation $> v_i$ the Galois-group acts on elements of valuation v_i via a tame character with values in k^*. This character is equal to the reduction of the Lubin-Tate character if and only if

$$v_i \equiv p^{-jh}/(p^h - 1) mod(\mathbb{Z})$$

for some integer $j \geq 0$ (For gr_0^F there is an easy relation between valuations and tame Galois-actions on leading terms. To check for gr_1^F one can use that the result is correct for the valuation of t, that is for valuation $1/(p-1)$, and that it is anyway correct up to a constant shift). For the i-th Frobenius transform one has to multiply the right hand side by p^i.

Now let us first study the case $d = 1$, that is Lubin-Tate groups. It then follows that the valuation v_i of x_i satisfies either $v_i > 1$ or $0 < v_i \leq p^i/(p^h - 1)$ $(0 \leq i < h)$. The first alternative cannot hold because x_i divides t which has valuation $1/(p-1)$.

Also the product of the x_i is a Galois-eigenvector for the cyclotomic character and thus a K_0^*-multiple of t. Checking valuations one finds that the factor is a unit, and that each x_i has valuation $v_i = p^i/(p^h - 1)$. Especially the determinant of the period map is the product of the x_i and thus up to a unit equal to t, as claimed. Also one checks that for each collection of $d < h$ indices i the product of the corresponding x_i has leading term with valuation < 1, thus must generate the corresponding Galois-eigenspace (which is a free cyclic module over V_0).

Now for general $d < h$ the corresponding elements x_i must be integral multiples of the product of d "Lubin-Tate-x_i's". Thus their product is an integral multiple of t^d. But this product is the determinant of the period-map, and applying the same procedure to the dual (d replaced by $h - d$) we obtain that the factor is a unit.

Remark: The result also applies to formal groups of finite height over $\hat{\bar{V}}$, the p-adic completion of the integral closure of V in an algebraic closure $\bar{K}$. Namely these can be obtained by pushout from a formal group over $V[[T]]$ (use [14] to check that modulo each power of p the formal group is defined over $\bar{V}$) and the previous deformation-argument applies.

3. Valuations of torsion-points

We fix a local field K which is a finite extension of $\mathbb{Q}_p$ and denote by $V = \mathcal{O}_K$ its ring of integers. p-adic valuations are normalised such that a uniformiser π of $\mathcal{O}_K$ has valuation 1. The order of the residue-field of V is denoted by q.

In this section we consider formal $\mathcal{O}_K$-modules H of dimension 1 and height h, as in [11], over the completion $\hat{\bar{V}}$. In a suitable coordinate T multiplication by π is described by a polynomial

$$f_H(T) = \pi T + \sum_{i=1}^{h-1} a_i T^{q^i} + T^{q^h},$$

where the coefficients $a_i \in \hat{\bar{V}}$ are non-units. They parametrise the (suitably defined) isomorphism-class of H. We say that H satisfies condition $(*)$ if a_i has valuation $v(a_i) \geq 1 - i/h$. It is shown in [11], Cor.23.26, that any H is isogeneous to one satisfying $(*)$.

The torsion-points in H form a module isomorphic to $(K/\mathcal{O}_K)^h$. The valuation of such a torsion-point is defined as that of the value of T at that point. The valuations of the π-torsion can be read of from the Newton polygon of f_H:

Namely in the (x, y)-plane form the convex hull of all points lying above the points $(q^i, v(a_i))$, $0 \leq i \leq h$ ($a_0 = \pi, a_h = 1$). Then its lower boundary consists of h lines connecting (q^{i-1}, u_{i-1}) to (q^i, u_i). If $v_i = (u_{i-1} - u_i/(q^i - q^{i-1})$ denotes the slopes of these lines then $H[\pi]$ has an $\mathbb{F}_q$-basis x_i such that for a linear combination $x = \sum_H r_i \cdot x_i$ ($r_i \in \mathbb{F}_q$) the valuation of x is equal to the minimal v_i for which $r_i \neq 0$. Furthermore the v_i are non-increasing.

Now assume that H satisfies $(*)$. We want to determine for which isogenies $f : H \to H'$ the target H' satisfies $(*)$ as well. We may assume that the isogeny f does not factor over multiplication by π, that is it does not annihilate all π-torsionpoints.

Lemma 2. *H' satisfies $(*)$ if and only if H' is the quotient of H under the submodule generated by $x_1, ..., x_i$, for some i for which $u_i = 1 - i/h$, and for a suitable choice of x_i (There is some ambiguity of several x_i have the same valuation).*

Proof: First assume that f is of the form above. It is known that for a point x of H the valuation of $f(x)$ is equal to the sum of the valuations $v(x -_H y)$, for y ranging over the elements of $ker(f)$ ("$-_H$" means difference in H). We apply this to either x a non-trivial combination of x_j with $j > i$, or to an x with $\pi \cdot_H x$ a non trivial combination of x_j with $j \leq i$. The images of these points generate the π-torsion in H'. From the Newton-polygon one checks that for $j \leq i$ our element has valuation v_i/q^h, and it follows in all cases that the valuation of $x -_H y$ ($y \in Ker(f)$) is equal to $v(x)$. Thus under f the valuation of x is multiplied by the order of f, that is by q^i. It follows that if we define $x'_j = f(x_{j+i})$ for $j \leq h - i$, and x'_j for $j > h - i$ as the image of an x with $\pi \cdot_H x = x_{j+i-h}$, then the H'-valuation of x_j is either $q^i \cdot v_{j+i}$ ($j \leq h - i$) or $q^{i-h} \cdot v_{j+i-h}$ (for $j > h - i$). As $v_1 \leq q^{h-1} \cdot v_h$ it follows that the sequence of these valuations is non-increasing, with a break at $h - i$, and that the valuation of any linear combination of the x'_j is equal to the infimum of the v'_j for which the coefficient is nonzero. One then can read of the parameters $u'_j = u_{j+i} - u_i$ respectively $u'_j = u_{j+i-h} + 1 - u_i$. As $u_i = 1 - i/n$ it follows that H' satisfies $(*)$.

Conversely assume that this is the case. We can find a basis ρ_i for the Tate-module $T_\pi(H)$ such that the x_i are the π-division-points defined by ρ_i, and such that the kernel of f is generated by the corresponding π^{n_i}-division points, for integers $n_i \geq 0$ not all strictly positive. That is $T_\pi(H')$ has basis $\pi^{-n_i} \cdot \rho_i$.

If n denotes the maximum of the n_i, choose elements y_i with $\pi^n \cdot_H y_i = x_i$ for those i for which $n_i = n$. Let r be the number of these, and denote them by $i_1 < i_2 < \ldots < i_r$. Any linear combination of the y_i not annihilated by π^n has valuation $\leq v_{i_0}/q^{nh}$. As elements $y \in Ker(f)$ have valuation $\geq v_h/q^{(n-1)h}$ it follows that under f the valuations of these elements multiply by the degree, that is by $q^{\sum n_i}$.

Now the image is a subgroup of the π-torsion of H', of rank r. It contains $(q-1)$ elements of valuation $v_{i_1}/q^{\sum(n-n_i)}$, $(q^2 - q)$ elements of valuation $v_{i_2}/q^{\sum(n-n_i)}$, etc.. The sum of these is equal to

$$
\begin{aligned}
&\Big(\sum_{j=1}^{r}(q^j - q^{j-1})v_{i_j}\Big)/q^{\sum(n-n_i)} \\
&\leq \Big(\sum_{j=1}^{r}(q^j - q^{j-1})v_j\Big)/q^{\sum(n-n_i)} \\
&\leq \Big(\sum_{j=1}^{r}(q^j - q^{j-1})v_j\Big)/q^{h-r} \\
&\leq (1 - u_r)/q^{n-r}.
\end{aligned}
$$

On the other hand as H' satisfies $(*)$ as well the sum must be $\geq r/(hq^{h-r})$. Thus $u_r = 1 - r/h$, $n = 1$, and $v_{i_j} = v_j$. Changing the basis x_i (it is not quite unique) gives the result.

Remark: The equality $u_i = 1 - i/h$ only happens of (q, u_i) is a proper vertex of the Newton polygon, that is a change in slopes occurs at that stage. Also this happens if and only if the π-adic valuation of a_i is $1 - i/h$.

Next we define a valuation (or a π-adic norm) on $T_\pi(H)^{dual}$. Namely consider the universal $\mathcal{O}_K$-vectorextension $E_K H$, as in [11]. The previous universal vectorextension maps to it, and it is the biggest quotient such that $\mathcal{O}_K$ acts on the

Lie-algebra via its natural homomorphism into $\hat{\bar{V}}$ (The Lie-algebra $M_1(G)$ of the usual universal vectorextension is a projective $\hat{\bar{V}} \otimes \mathcal{O}_K$-module). An element ρ of $T_\pi(H)$ is given by a sequence $x_n = \pi \cdot_H x_{n+1}$ of torsion-elements. Lift them to $E_K H$ and multiply by π^n. The result is modulo $\pi^n \cdot F_K(\hat{\bar{V}})$ independant of choices, and the limit defines a lift $T_\pi(H) \to E_K H(\hat{\bar{V}})$ (Which is also the lift given by the crystalline nature of universal vector-extensions). In fact this construction even defines a lift on the vectorspace $T_\pi(H)[1/\pi]$.

The Tate-module $T_\pi(H)$ maps into the vectorpart F_K, and by duality we get $F_K^{dual}(\hat{\bar{K}}) \to T_\pi(H)^{dual} \otimes_{\mathcal{O}_K} \hat{\bar{K}}$. The quotient is one dimensional and induces the norm on $T_\pi(H)^{dual}$ (that $T_\pi(H)^{dual}$ injects into it will be seen below). Another way: Lift H to an $\mathcal{H}$ over $A_{crys}(V)$. The first crystalline homology $M_1(\mathcal{H})$ is independant of the choice involved. Consider the period-map

$$T_\pi(H) \to M_1(\mathcal{H}).$$

Further divide by $M_1(\mathcal{H}) \otimes F^1(A_{crys}(V))$ and map to the maximal quotient of $M_1(H)$ on which $\mathcal{O}_K$ acts via its embedding into $\hat{\bar{V}}$. The resulting map takes values in $F_K(\hat{\bar{V}})$ and is the same as before.

It follows by duality that the one-dimensional quotient has the following description: Consider the dual (p-divisible or formal group) H^{dual}. It has height $h \cdot [K : \mathbb{Q}_p]$ and dimension one less than that. It admits an action of $\mathcal{O}_K$ but is not an $\mathcal{O}_K$-module since the action on the Lie-algebra is not correct. Its universal vector-extension has a one-dimensional vectorpart dual to the Lie-algebra of H, on which $\mathcal{O}_K$ acts by multiplication. The previous construction applied to H^{dual} defines a map

$$T_p(H^{dual}) = T_p(H)^{dual}(1) \to Lie(H)^{dual}(\hat{\bar{V}})$$

which (up to a factor) is our quotient-map. That it is injective follows from

Lemma 3. *Suppose that modulo p H is isogeneous to H_0 of order p^n, that is there exist morphisms in both directions with composition multiplication by p^n. Then the induced p-adic valuation on $T_p(H^{dual})$ differs from the standard-valuation by a function taking values between 0 and $n+1$.*

Proof: Consider an indivisible element $\rho \in T_p(H^{dual})$ and suppose its image in $Lie(H)^{dual}(\hat{\bar{V}})$ is divisible by p^{n+1}. This means that the corresponding p^{n+1}-torsionpoint x of H^{dual} lifts to a p^{n+1}-torsion-point in EH^{dual}. By the crystalline nature of the universal vectorextension the isogeny modulo p between H and H_0 lifts to an isogeny over $\hat{\bar{V}}$ between universal vectorextensions, again of order p^n. Thus EH_0^{dual} must contain a nontrivial torsion-point of exact order p. As we have a Lubin-Tate group all such torsion-points are conjugate under automorphisms, so the period-map for H_0^{dual} vanishes modulo p. On the other hand we have already studied the periods of such formal groups, and know that the period divides t with the quotient a non-unit. Thus the period has p-valuation < 1 and is not divisible by p.

Remark: This argument partially extends to families, using the methods from [6]: Suppose G is a formal group over a p-adically complete ring R which is a V-algebra and has toroidal singularities over V (for example is smooth, or has semistable reduction, see [9]). If $\bar{R}$ denotes its normalisation in the maximale étale cover of $R[1/p]$ then $\bar{R}$ is almost flat over R, that is all higher Tor's are annihilated by any positive power of p (see [9] for details). Also the relative (logarithmic) differentials $\Omega_{\bar{R}/R}$ are almost isomorphic to a direct sum of copies of $\bar{R}[1/p]/R$, with one direct summand isomorphic to $\Omega_{\bar{V}/V} \otimes_{\bar{V}} \bar{R}$. Finally the logarithmic derivatives of roots of unity define an isomorphism

$$\bar{V}[1/p]/\eta^{-1} \cdot \bar{V} \cong \Omega_{\bar{V}/V},$$

for a certain $\eta \in V$ (whose valuation is related to the discriminant of V). It then follows from lemma 2 in [6] that the Hodge-tate period map has an inverse up to η (first an almost inverse, but an almost integral element of $\hat{\bar{R}}[1/p]$ is integral). Thus the submodule of $Lie(G)^{dual} \otimes \hat{\bar{R}}$ generated by the image of $T_p(G^{dual})$ contains anything divisible by η. Especially over the field it spans the first stage of the Hodge-filtration.

4. Two Moduli-spaces

V.G.Drinfeld has studied two different moduli-spaces. One of these is his famous symmetric space, namely the complement of all K-rational hyperplanes in $\mathbb{P}_K^{h-1}$. It has an integral model X (a formal scheme locally of finite type over the maximal unramified extension of $\mathcal{O}_K$) which is the Deligne-scheme: Its R-points (R an artinian local $\mathcal{O}_K$-algebra on which p is nilpotent) associated to any lattice $\Lambda \subset K^h$ a quotient line $\Lambda \otimes R \twoheadrightarrow \mathcal{L}_\Lambda$, invariant under homotheties. Furthermore inclusions of Λ's should induce maps on the quotient, and for any nontrivial $\rho \in K^h$ there exists a Λ such that the intersection of Λ and the line spanned by ρ generates $\mathcal{L}_\Lambda$. X classifies formal groupes G of dimension h with multiplication by $\mathcal{O}_B$, equipped with a quasiisogeny modulo p (isogeny up to inverting a power of π) of degree 1 to a fixed G_0, and such that the integers of the unramified extension of degree h of K act via the regular representation on the Lie-algebra of G (special $\mathcal{O}_B$-module in [4]). Choose a uniformiser Π of $\mathcal{O}_B$ with $\Pi^h = \pi$ and such that Π normalises the integers of the unramified extension of degree h of K which are contained in B.

Also there exists finite coverings $X_n \to X$ where X_n classifies G's (as before) together with a generator of its Π^n-division points. Here a section $g \in G$ is such a generator if $\Pi^{n-1} \cdot g$ is a generator of the Π-division points, which means that it lies in a certain closed subscheme which is functorial, and which in characteristic zero coincides with the complement of the zero-section in $G[\Pi]$. All in all we obtain a proobject in the category of formal schemes, with flat transition-maps. We denote by X_∞ its projective limit in the category of π-adic formal schemes, that is the projective limit of the topological spaces, endowed with the completion of the inductive limit of sheaves of rings. It admits an action by the subgroup of $GL(h,K) \times B^* \times W_K$ consisting of elements where the sum of π-adic valuations of determinants vanishes (on W_K any Frobenius has valuation 1). The action is by changing the quasi-isogeny by isogenies ($GL(h,K)$), elements of B^* (where one also has to conjugate the $\mathcal{O}_B$-action on G), and by composing it with powers of the Frobenius-isogeny on G_0. It is shown in [17] 5.48 that the period map into projective space defined by Drinfeld coincides with the crystalline period map.

Similarly in [11] we have the universal deformation-space of the formal $\mathcal{O}_K$-module of dimension 1 and height h. Rigid-analytically it corresponds to the open unit disk. It specifying Drinfeld-basis of π^n-torsion points one obtain a flat cover which represented by a regular complete local ring of dimension h. The coordinates of the universal division-points give regular parameters (see [13]). In the limit we obtain an infinite covering with group $GL(n, \mathcal{O}_K)$. On it the bigger group $GL(n, K)$ acts: The scalar π acts trivially. Suppose we are given H with a compatible system of Drinfeld-basis for its division-points, and a $g \in GL(h, K)$. Multiply g by a power of π to make it integral, and choose n sufficiently big such that g divides π^n. The linear combinations of the g-transforms of the Drinfeld-basis of $H[\pi^n]$ generate a flat subgroup whose underlying scheme is the divisor which is the sum of all linear combinations. Dividing by this subgroup gives a new H with Drinfeld level-structures. Over the closed point the induced isogeny is isomorphic to a power of Π (Π is a uniformiser of $\mathcal{O}_B$, and the exponent is the valuation of $det(g)$), so the special fibre of the quotient is again isomorphic to the fixed H_0.

More generally we can find a model over the maximal unramified extension of $\mathcal{O}_K$, with an action of the familiar subgroup of $GL(h, K) \times B^* \times W_K$ of elements with sums of valuations 0. Here the first factor acts via the inverse adjoint of the previous, the second and third by quasi-isogenies on H_0 (the inverse adjoint is needed because the first factor now acts on H instead of H_0).

We now construct formal p-adic schemes mapping to these, corresponding to an affinoid covering of the projective system of open unit disks. One such domain is the following: Consider the closed unit disk classifying H's for which the coefficients a_i of the polynomial f_H have valuation $\geq 1 - i/h$. If we adjoin an h'th root of π it has a smooth formal model, namely the formal affine space in coordinates $a_i/\pi^{1-i/h}$. There is also a natural model over $\mathcal{O}_K$, whose affine ring is the intersection with $K[[a_1, ..., a_{h-1}]]$, but it is not smooth (only toroidal), and our affine space is the normalisation of its base-extension. Over these models we have the universal formal group H (where $f_H(T)$ has coefficients a_i), and a $GL(h, \mathcal{O}_K)$-covering defined by Drinfeld level structures. This covering is first a pro-object in the category of p-adic formal schemes but we may form its projective limit as a p-adic formal scheme. It is affine with algebra the π-adic completion of the union of the algebras classifying π^n-levelstructures.

Next form a disjoint union of these indexed by $GL(h, K)/\pi^{\mathbb{Z}} \cdot GL(h, \mathcal{O}_K)$, and glue them along open subschemes, according to the following rules (dictated by lemma 2):

Everything should by $GL(h, K)$-equivariant, so it is enough to specify how the copy parametrised by $g = 1$ is glued (and this has to be $GL(h, \mathcal{O}_K)$ equivariant).

So for this copy consider the open formal subscheme defined by $a_i/\pi^{1-i/h}$ invertible. Over it divide H by the flat subgroup consisting of π-torsion-points of valuation $\geq i/(h \cdot (q^i - 1))$. It has order q^i and is defined by the vanishing of a suitable factor of f_H. The result H' lies again in our fundamental region, that is $a'_j/\pi^{1-j/h}$ is integral (and invertible for $j = h - i$). Also Zariski-locally H' admits a Drinfeld level structure, by composing the one on H with a suitable matrix g which is a product of an element of $GL(h, \mathcal{O}_K)$ and the inverse of $diag(\pi, \pi ..., 1, 1)$ (i π's). Glue our open subscheme to the corresponding open in the copy defined by g.

The result of this glueing is a π-adic formal scheme Y_∞. Over it we have a one-dimensional formal $\mathcal{O}$-module H, together with a quasiisogeny modulo π, of degree 0. If one wants to use pro-objects the result is somehow difficult to describe. It is someting like an increasing union of proobjects in the category of formal π-adic schemes. It is easier to form the quotient under a compact open subgroup $M \subset GL(h, K)$, which is a π-adic formal scheme Y_M over $\mathcal{O}_K$, locally of finite type. The Y_M form a projective system, with an action of $GL(h, K)/\pi^{\mathbb{Z}}$. Also on everything acts the subgroup of $GL(h, K) \times B^* \times W_K$ of elements of "unit-determinant".

Finally the rigidanalytic space defined by Y_∞ maps via the a_i to the open polydisk. If we restrict to the preimage of a closed polydisk, say defined by the condition that all a_i have valuation $\geq \epsilon > 0$, then the proof of corollary 23.26 in [11] shows that the H involved are isogeneous to an H' satisfying $(*)$ such that the degree of the isogeny is bounded in terms of ϵ (First bound the valuations of π-torsionpoints away from 0). Thus the preimage lies in a finite union of transforms of the fundamental region. Hence Y_∞ defines a model for the infinite rigid covering of the open unit disk defined by an infinite levelstructure on H.

Finally there exists another "period-mapping", as in [11], section 23: Rigid-analytically it maps H to the point in $\tilde{P} = \mathbb{P}^{h-1}$ which classifies the Hodge-filtration of $Lie(EH_0)$ defined by H. After adjoining an h-th root of π $\tilde{P}$ has a model with homogeneous coordinates $b_i/\pi^{1-i/h}$. The b_i are (as the a_i) eigenvectors for the units of the unramified extension of degree h of K considered as subgroup of B^*. The uniformiser Π acts by cyclically permuting the $b_i/\pi^{1-i/h}$ $(b_h = b_0)$, see [11], equ. 22.9. On the fundamental domain (a_i valuation $\leq 1-i/h$) the period mapping is of the form $b_i = a_i + higherorder$. It extends to a regular map $Y_\infty \to \tilde{P}$ which is equivariant under the subgroup of elements with unit determinant in $GL(h, K) \times B^*$, (where the first factor acts trivially on $\tilde{P}$).

We remark that these construction have been done over the ring V obtained by adjoining to $\mathcal{O}_K$ an h'th root of π. However as the constructions are independant from the choice of such a root we get natural singular models over $\mathcal{O}_K$. Fortunately we do not need these: It suffices that our models satisfy a "weak descent-condition", namely that over the normalisation of $V \otimes_{\mathcal{O}_K} V$ the two pullbacks become isomorphic, the isomorphism satisfying transitivity etc.. This "weak descent" will be sufficient for our applications to vanishing cycles.

5. Relation

We prove that after suitable blowups the two infinite covers X_∞ and Y_∞ become isomorphic. We shall construct such an isomorphism over the maximally unramified extension of the extension of $\mathcal{O}_K$ defined by adjoining an h'th root of π, and it will satisfy the "weak descent condition" as before, that is nothing really depends on the choice of that root.

We first describe this correspondence on the level of $\hat{\hat{V}}$-points where the blowups do not show up. We denote by H_0 a lift of the $\mathcal{O}$-module of dimension 1 and height h, say the one defined by $a_i = 0$. Modulo π it has multiplication by $\mathcal{O}_B$, where Π acts as Frobenius. The universal vectorextension EH_0 is up to isogeny independant of the lift. Also over k there exists a special $\mathcal{O}_B$-module G_0 and an isogeny from H_0^h into it, of degree $q^{h(h-1)/2}$. Namely one can choose for G_0 the product of h copies of H_0, with the isogeny on the i-th factor given by Π^{i-1}, and the corresponding conjugation of the $\mathcal{O}_B$-action. Thus the Drinfeld halfspace describes $\mathcal{O}_B$-modules G with a quasiisogeny module π $H_0^h \to G$ of degree $q^{-h(h-1)/2}$.

Now assume given such a G defined over $\hat{\hat{V}}$, together with a compatible system of generators of its Π^n-division-points. Then EG is isogeneous to EH_0^h, and the division-points define injections of B into the $\hat{\hat{V}}$-points of EG and EH_0^h. The logarithm of $1 \in B$ defines an element $x = (x_1, ..., x_h) \in M_1(H_0)^h \otimes \hat{\hat{V}}[1/p]$. Its B-span is a B-stable subvectorspace of codimension h, thus induced from a subspace

$F_G \subset \hat{\hat{V}}[1/\pi]^h$ of codimension 1 which is the classifying point associated to G in the Drinfeld symmetric space. By duality x defines a map $\hat{\hat{V}}[1/\pi]^h \to M_1(H_0) \otimes \hat{\hat{V}}[1/\pi]$ whose image F_H has codimension 1. It defines a point in $\tilde{\mathbb{P}}$ and thus an object H "up to isogeny". First choose one such $\tilde{H}$ such that modulo π there exists an isogeny $H_0 \to \tilde{H}$, and such that under the lift $EH_0 \to E\tilde{H}$ the elements $x_1, ..., x_h \in M_1(H_0) \otimes \hat{\hat{V}}[1/\pi]$ map to the integral elements in the vector-part of $Lie(E\tilde{H})$ (multiply the isogeny by a high power of π). Then x defines a map $\mathcal{O}_K^h \to T_\pi(\tilde{H})$ whose degree can be computed using Lemma 1:

Namely $\mathcal{O}_B \cdot x$ is the Tate-module of G, and the period-map for G has determinant t^h. Up to a factor $q^{-h(h-1)/2}$ this is the determinant of the map from $\mathcal{O}_B$ into $Lie(EH_0) \otimes A_{crys}^h[1/p]$, which in turn is $q^{h(h-1)/2}$ times the h-th power of the determinant of the $\mathcal{O}_K$-submodule generated by $x_1, ..., x_h$ in $Lie(EH_0) \otimes A_{crys}(V)[1/p]$ (see below). Thus this has (up to units) determinant t, and the map above on Tate-modules has degree equal to that of the isogeny (modulo π) $H_0 \to \tilde{H}$. It follows that there is an H over $\hat{\hat{V}}$ with an isogeny $H \to \tilde{H}$ such that $x_1, ..., x_h$ is a basis of $T_\pi(H)$ and such that H has special fibre H_0. Furthermore the x_i define a Drinfeld level-structure on H.

We have used implicitely that the degree of an isogeny can be read of from Tate-modules (which live in the general fibre) as well as from the crystalline homology (which depends only on the fibre in the residue-field of $\hat{\hat{V}}$). For the remaining assertion above about determinants of $\mathcal{O}_B$-stable lattices replace $\mathcal{O}_K$ by a finite unramified extension which splits B and $\mathcal{O}_B$ by the $h \times h$-matrices over that extension (this leads to the factor $q^{h(h-1)/2}$). Then all modules over this matrixring are just sums of h copies of a module over the base, and everything becomes elementary.

Conversely given an H with Drinfeld level-structure, that is a basis $x_1, ..., x_h$ of its Tate-module, lift them to $Lie(EH_0)$ and consider the B-submodule generated by $x = (x_1, ..., x_h)$ in $Lie(EH_0)^h$. It is induced from a subspace F_G which lies in the Drinfeld halfspace (use lemma 3) and corresponds to a G. Also x determines a level-structre up to a power of Π, and comparing determinants shows that this power is 0. Finally it is clear that these correspondences are inverse to each other. The relevant object is $x \in Lie(EH_0)^h \otimes \hat{\hat{V}}[1/\pi]$ which determines all the rest.

To see that this bijection on $\hat{\hat{V}}$-points comes from an isomorphism of formal schemes consider the following blow-ups: Over Y_∞ the universal period $T_\pi(H) \to F$ defines $T_\pi(H)^{dual} \to Lie(H)^{dual}$. Zariski-locally we have that for any indivisible $\rho \in T_\pi(H)^{dual}$ its image divides a fixed π-power π^n: Namely it suffices to check that over the open affine where H satisfies $(*)$. There H is modulo p isogeneous to a fixed H_0, of degree bounded by p^n, and then lemma 3 implies the assertion for all $\hat{\hat{V}}$-points. If U_{N+1} then denotes the affine classifying H satisfying $(*)$ and a level π^{N+1}-structure, the image of ρ modulo π^{N+1} is defined over U_{N+1}. Lifting it to a regular section f of $Lie(H)^{dual}$ over U_{N+1} we get from rigid analysis that this section does not vanish on the generic fibre (a rigid space) and thus must divide a power of π. Then π^N/f is integral over $\mathcal{O}_{U_{N+1}}$. As the image of ρ coincides with f modulo π^{N+1} it follows that in the extension of $\mathcal{O}_{U_\infty}$ generated by the normalisation of $\mathcal{O}_{U_{N+1}}$ the image is of the for $u \cdot f$ with $u \equiv 1 mod(\pi)$. Thus u is a unit, so the image of ρ divides π^{N+1} in this extension. Multiplying with an other π-power which sends the normalisation of $\mathcal{O}_{U_{N+1}}$ into $\mathcal{O}_{U_{N+1}}$ we get the assertion. Now $\tilde{Y}_\infty$ is obtained from Y_∞ by blowing up the ideal generated by the image of

Λ, for any lattice $\Lambda \subset K^h$. We claim that it suffices to blow up only finitely many ideals and that they are already defined over Y_n: Namely we need only consider Λ's up to homothety, so we may assume that they lie in $T_\pi(H)^{dual} = \mathcal{O}_K^h$ and that they contain an indivisible ρ as above. But then two Λ's generate the same ideal if they coincide modulo π^n, thus the finiteness of the number of ideals. Also all ideals contain π^n and are modulo that generated by elements in $\mathcal{O}_{Y_n}$.

For the definition of the appropiate modification $\tilde{X}_\infty$ we note that over X_∞ the universal torsion-point defines a period-map $\mathcal{O}_B \to Lie(G)^{dual}$ which after inverting π define a map into $\tilde{P}$, and this becomes regular after blow-up of a suitable ideal. By the remark after lemma 3 on sees that the ideal is defined at a finite level.

Finally from $\tilde{Y}_\infty$ we have a regular map into Drinfeld's model X_0, thus an $\mathcal{O}_B$-module G over it. EG is quasiisogeneous to EH_0^h and EH^h, and the lifts of the universal division-points define a map $K \to EG$ and thus a B-linear $\mathcal{O}_B \to EG \to G$. Over each $\hat{\bar{V}}$-point this defines a level-structure, so a level structure in general, and thus a map from $\tilde{Y}_\infty$ to X_∞. This map factors over $\tilde{X}_\infty$ because $\tilde{Y_\infty}$ maps to $\tilde{P}$, that is the relevant ideals are already invertible in it.

Conversely starting with $\tilde{X}_\infty$ it maps to $\tilde{P}$. Thus over the preimage of any standard affine in $\tilde{P}$ we obtain by [11], Cor.23.15 a $\tilde{H}$, and a map $K^h \to E\tilde{H} \to \tilde{H}$. Its kernel $T_\pi(\tilde{H})$ is an $\mathcal{O}_K$-lattice which is Zariski-locally constant, and contained in $\pi^{-n} \cdot \mathcal{O}_K^h$ with index q^{nh} for sufficiently big n, as can be checked on $\hat{\bar{V}}$-points. Choosing a basis for $T_\pi(\tilde{H})$ defines a Drinfeld-levelstructure and thus a map into Y_∞, which lifts into $\tilde{Y}_\infty$ because the blowups necessary to map into X_0 are trivial here. Transforming with a suitable element of $GL(h, K)$ with determinant a unit we obtain the desired H with infinite level-structure ($\mathcal{O}_K^h \cong T_\pi(H)$). This gives the inverse $\tilde{X}_\infty \to \tilde{Y}_\infty$. It is clear that it is compatible with the action of the subgroup of $GL(h, K) \times B^* \times W_K$ of elements with "unit-determinant".

6. Consequences for nearby cycles

For an integer r prime to p it is explained in [1] how to define the sheaves of vanishing cycles $R^i\Psi_\eta(\mathbb{Z}/(r))$ on the special fibres of $\tilde{X}_n$ and $\tilde{Y}_n$. These coincide with the vanishing cycles defined by any algebraic model. The inductive limit defines vanishing cycles also on the special fibers of $\tilde{X}_\infty$ and $\tilde{Y}_\infty$, and these are isomorphic (using [1], Th. 7.1). Furthermore by [2], Cor.2.5 the cohomology with compact support $R\Gamma_c(Y_{n,s}, R\Psi_\eta(\mathbb{Z}/(r)))$ coincides with the local cohomology with support in the closed point $R\Gamma_x(R\Psi_\eta(\mathbb{Z}/(r)))$, for any algebraic $\mathcal{O}_K$-scheme whose formal completion in the point x equals the formal scheme which represents deformations H of H_0 together with a Drinfeld level-π^n-structure (This formal scheme is represented by a regular complete local ring). Especially these are finite.

Passing to the limit we obtain the cohomology with compact support of the vanishing cycles on X_∞, $\tilde{X}_\infty$, Y_∞, and $\tilde{Y}_\infty$. They admit continuous actions of the subgroup of "elements of determinant a unit" of $GL(h, K) \times B^* \times W_K$, and the submodule stabilised by any open compact subgroup in $GL(h, K)$ is finite. It is more convenient to consider the induced representation of $GL(h, K) \times B^* \times W_K$ modulo the subgroup generated by $(\pi, 1, 1)$, as in [7].

Finally for l-adic cohomology we form the projective limit of cohomologies mod l^s, and in it the elements stabilised by an open compact subgroup in $GL(h, K)$ which is a pro-p-group. We get the same result if we first form the projective limit at some finite level (that is form the l-adic cohomology of some Y_n which is finite over $\mathbb{Z}_l$ as seen above, not for general reasons), and then the direct limit over all levels. Finally inverting l gives $\mathbb{Q}_l$-adic cohomology.

It is shown in [13] that the result realises the Jacquet-Langlands correspondence as well as the local Langlands-conjecture: Namely a given supercuspidal representation σ of $GL(h,K)$ occurs with finite multiplicity and the alternating sum of the multiplicities is $(-1)^{h-1} \cdot h$. Furthermore in the Grothendieck-group of representations the eigenspace for the dual representation σ^{dual} is equivalent to the tensorproduct of the associated (local Jacquet-Langlands) representation $JL(\sigma)$ of B^*, and the h-dimensional representation of W_K predicted by the local Langlands-correspondence. Our main result now implies that similar assertions hold as well for the cohomology of (the coverings of) the Drinfeld upper halfspace (no more duality because the two actions of $GL(h,K)$ differ by the outer automorphism ${}^tg^{-1}$ which sends σ to its dual, as can be checked on traces using that ${}^tg$ is conjugate to g).

Remark: Without too much effort one can improve a little bit on [13], namely one shows that the σ-eigenspace occurs only in cohomology in degree $h-1$. For the Drinfeld symmetric space this has been done by M.Harris in [12], for the other space it is at least known to the authors of [13]. One possible argument goes as follows:

[13] constructs a Shimura-variety (of dimension $h-1$) whose completion at a closed point is the deformation-space for H_0. For our purpose we fix suitable level-structures away from π and denote by Sh_n the quotient associated to a Drinfeld level-π^n-structure. It classifies abelian varieties A with certain endomorphisms and polarisations, and their local deformations are classified by a certain one-dimensional π-divisible direct summand $H \subset A[\pi^\infty]$ (the latter is essentially the sum of H and its dual). The special fibers $\bar{Sh}_n$ have stratifications according to the rank of the maximal étale quotient of H. Let $\bar{Sh}_n^{\,i}$ denote the locally closed reduced subscheme where this rank is $h-i$. It has pure codimension i, and our deformation-spaces for H_0 arise as formal completions at points in $\bar{Sh}_n^{\,h-1}$. For simplicity we denote by Sh the pro-scheme given by the projective system Sh_n. It admits an action of $GL(h,K)$.

Over the formal completion of Sh_n along $\bar{Sh}_n^{\,i}$ the π-divisible group H becomes an extension

$$H^{inf} \to H \to H^{et},$$

with H^{inf} formal of dimension 1 and height i, and H^{et} étale of height $h-i$. There is an open and closed subscheme $\bar{Sh}_n^{\,i,0} \subseteq \bar{Sh}_n^{\,i}$ classifying level-structures for which the first i generators lie in H^{inf}.

For the projective system we obtain similarly $\bar{Sh}^{i}$ and $\bar{Sh}^{i,0}$. They are stable under $GL(h,K)$ respectively $P_i(K)$, where $P \subset GL(h,K)$ denotes the parabolic stabilising a subspace of dimension i. In fact the formal completion along $\bar{Sh}^{i}$ is the formal $GL(h,K)$-scheme induced (or better produced) from the formal $P_i(K)$-scheme "completion along $\bar{Sh}^{i,0}$". It follows that the cohomology of $\bar{Sh}^{i}$ with values in the vanishing cycles of $\bar{Sh}$ is induced from the $P_i(K)$-representation on the cohomology of $\bar{SH}^{i.0}$. If we show that the unipotent radical $N_i(K)$ of $P_i(K)$ acts trivially on any irreducible subquotient in this cohomology the induced representation has (for $i > 0$) no supercuspidal subspace.

So suppose given an $u \in N_i(K)$. It has some denominator π^m, and induces maps $Sh_{n+m} \to Sh_n$. We first show that it acts trivially (that is the same way as $u = 0$) on closed points of $\bar{Sh}^{i,0}_{n+m}$), and then also trivially on stalks of vanishing cycles.

An R-point of the formal completion of Sh_{n+m} along $\bar{Sh}^{i,0}_{n+m}$ is given by an abelian variety A over R, with certain additional data which include a π-divisible extension

$$H^{inf} \to H \to H^{et},$$

and π^{n+m}-division points $e_1, ..., e_i \in H^{inf}$, $f_{i+1}, ..., f_h \in H$ which form Drinfeld-basis for H^{inf} and H^{et}, respectively. u maps $\pi^m \cdot f_j$ to elements $\tilde{f}_j$ which are the sum of $\pi^m \cdot f_j$ and a linear combination of the e's. The u-image is defined by dividing H by the submodule (of order q^{mh} generated by the $\pi^m \cdot e_j$ and $\tilde{f}_j$. As usual the scheme structure is that of the divisor which is the sum of all points. Also the level-structure on the quotient is defined by the images of e_j and $\pi^{-m} \cdot \tilde{f}_j$ (these are welldefined modulo the previous). Of course all this translates into welldefined operations on the level of abelian varieties.

Now over any algebraically field k of characteristic p all $e_i = 0$, and u operates trivially on k-points. For the action on stalks of vanishing cycles it depends by [2], Th.3.1 only on the formal completion along the relevant point. This formal completion describes deformations of H as above, and can be analysed as follows:

The formal π-divisible module H_{inf} with π^{n+m}-levelstructure is parametrised by a local ring of relative dimension $i - 1$, as before. The extension above is given by chosing $h - i$ sections $\tilde{f}_j \in H^{inf}$, and then H is the quotient of $H^{inf} \times K^{h-i}$ under the $\mathcal{O}_K$-submodule generated by $\pi^n \cdot (\tilde{f}_j, 1)$. Finally the images f_j of $(\tilde{f}_j, 1)$ define the levelstructure.

It follows that the formal completion is formally smooth over the moduli-space classifying H^{inf} with its levelstructure. Thus the vanishing cycles are induced from this, and as u respects the projection it acts trivially on them.

Now coming back to the Shimura-variety we see that the cohomology with compact support of $\bar{Sh} - \bar{Sh}^{h-1}$ has no supercuspidal subquotient, and by duality the same holds for the cohomology without support. Thus for a given supercuspidal σ the σ component in the cohomology of Sh (that is the cohomology of $\bar{Sh}$ with values in the vanising cycles) coincides with the σ-component in either the restriction of the vanishing cycles to $\bar{Sh}^{h-1}$ or in the cohomology with support in that subscheme. From the first description we know that it occurs only in cohomological degrees $\leq h - 1$, from the second in degrees $\geq h - 1$ (by duality). Combining we get the result.

Thus in the cohomology of X_∞ or Y_∞ supercuspidal representations occur only in the middle degree $h - 1$. The corresponding space is the product of the associated representation $JL(\sigma^{dual})$ or $JL(\sigma)$ and an h-dimensional representation of W_K. The hard content of [13] is that this representation realises the local Langlands-conjecture.

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