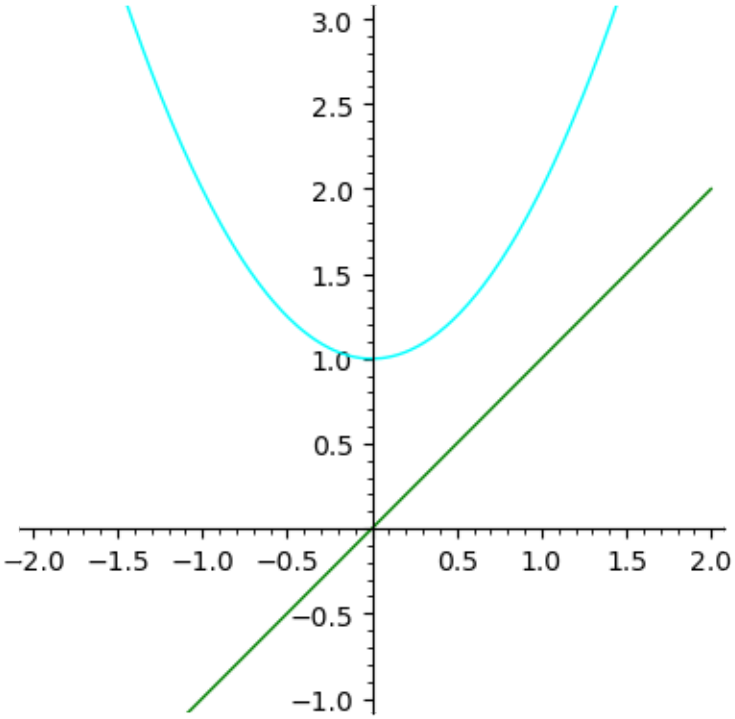
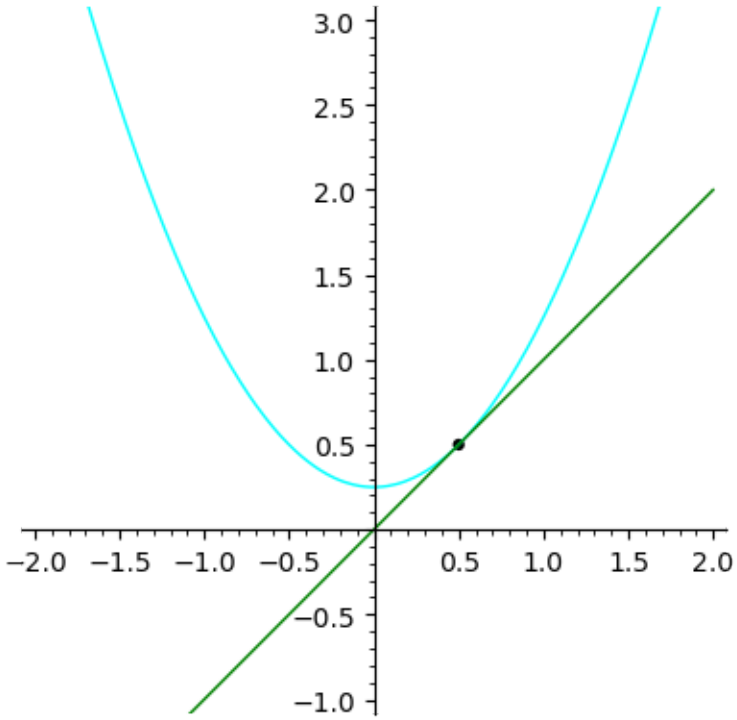
mandelbrot_intro_I

May 25, 2021

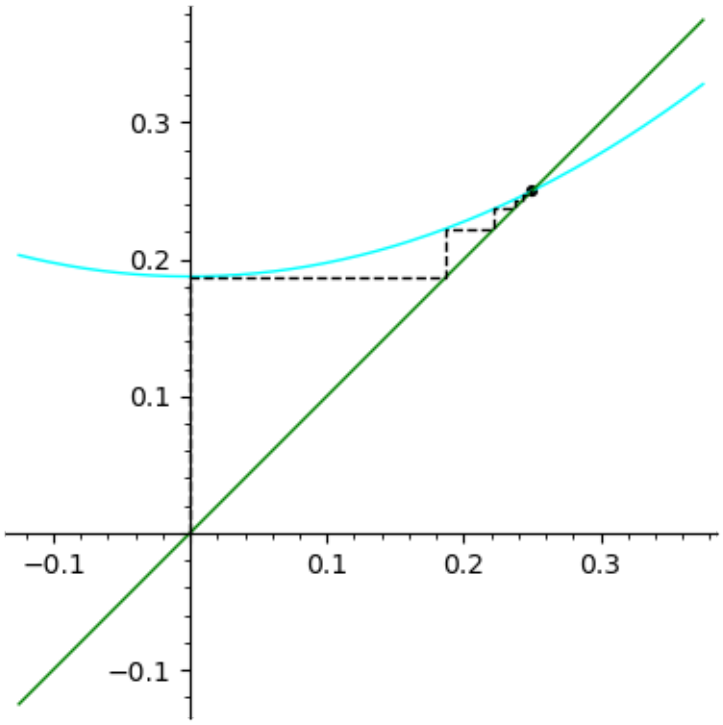
[2]:
```
R.<c> = CC['c']
f = x^2 + c
pf = plot(f(c = 1), (x, -2, 2), color = 'cyan')
d = x
pd = plot(d, (x, -2, 2), color = 'green')
(pf + pd).show(aspect_ratio = 1, xmin = -2, xmax = 2, ymin = -1, ymax = 3)
```



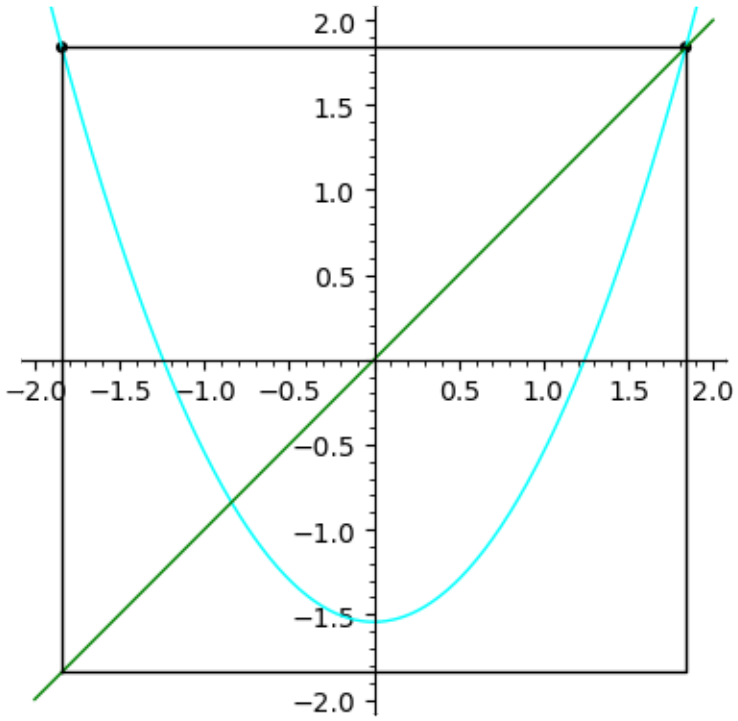
[3]:
```
pf = plot(f(c = 1/4), (x, -2, 2), color = 'cyan')
p = list_plot([[1/2, 1/2]], color = 'black', size = 20)
(p + pf + pd).show(aspect_ratio = 1, xmin = -2, xmax = 2, ymin = -1, ymax = 3)
```



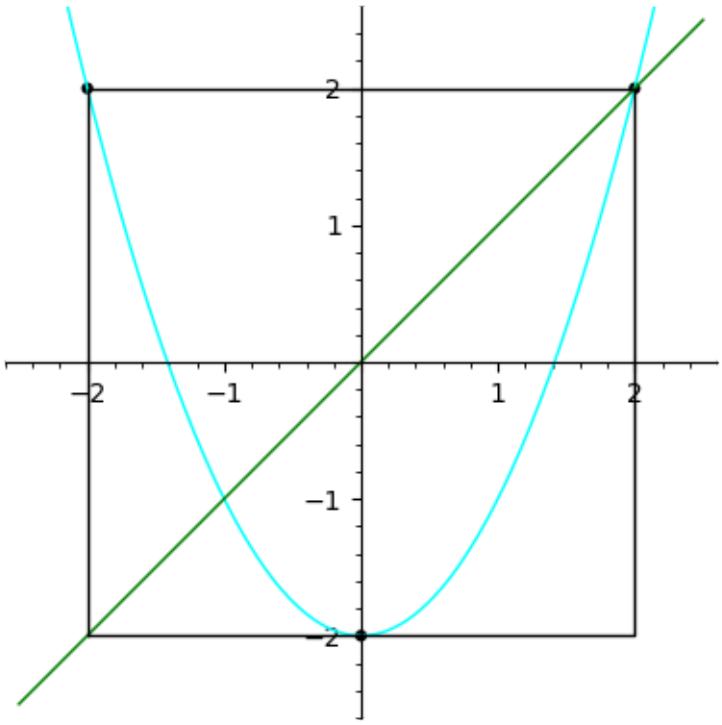
[4]:
```
c1 = 3/16
pf1 = plot(f(c = c1), (x, -1/8, 3/8), color = 'cyan')
pd1 = plot(d, (x, -1/8, 3/8), color = 'green')
p1 = list_plot([[1/4, 1/4]], color = 'black', size = 20)
it = 0
orbit = [(0,0)]
for i in range(20):
    nit = f(c = c1, x = it)
    orbit.append([it,nit])
    orbit.append([nit,nit])
    it = nit
porbit = list_plot(orbit, plotjoined = True, linestyle = '--', color = 'black')
(pf1 + pd1 + p1 + porbit).show(aspect_ratio = 1, xmin = -1/8, xmax = 3/8, ymin␣
↪= -1/8, ymax = 3/8)
```



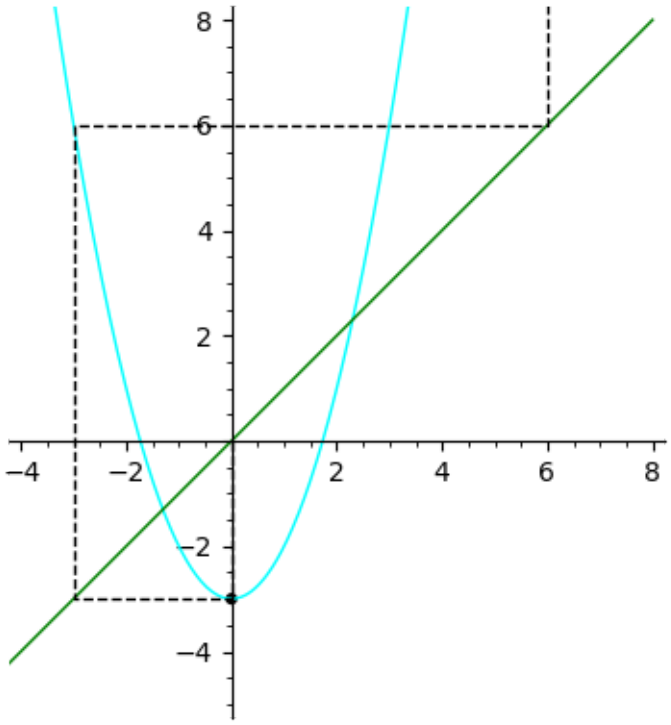
[5]:
```
s = 2
c0 = -1.54368901269208
pf = plot(f(c = c0), (x, -s, s), color = 'cyan')
pd = plot(d, (x, -s, s), color = 'green')
b = 1.83928675521416
sq = list_plot([(-b, -b), (-b,b), (b,b), (b, -b), (-b, -b)], plotjoined = True,␣
↪color = 'black')
pts = list_plot([(-b, b), (b, b)], color = 'black', size = 20)
(pf + pd + sq + pts).show(aspect_ratio = 1, xmin = -s, xmax = s, ymin = -s,␣
↪ymax = s)
```



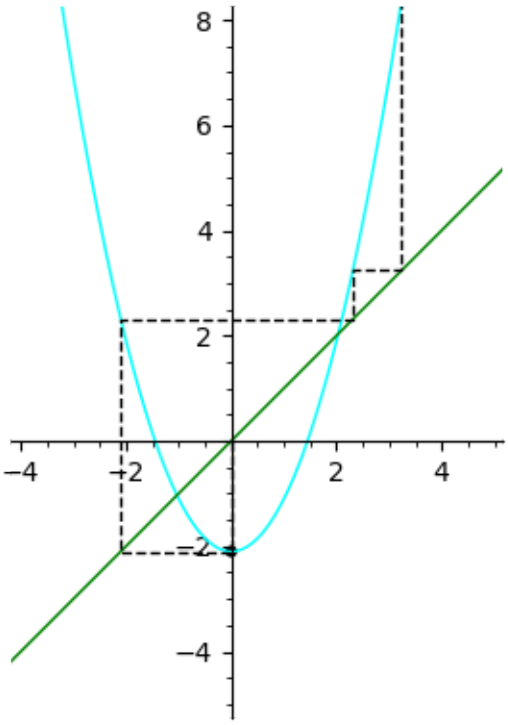
[6]:
```
s = 2.5
pf = plot(f(c = -2), (x, -s, s), color = 'cyan')
pd = plot(d, (x, -s, s), color = 'green')
sq = list_plot([(-2, -2), (-2,2), (2,2), (2, -2), (-2, -2)], plotjoined = True,␣
 ↪color = 'black')
pts = list_plot([(-2, 2), (2, 2), (0, -2)], color = 'black', size = 20)
(pf + pd + sq + pts).show(aspect_ratio = 1, xmin = -s, xmax = s, ymin = -s,␣
 ↪ymax = s)
```



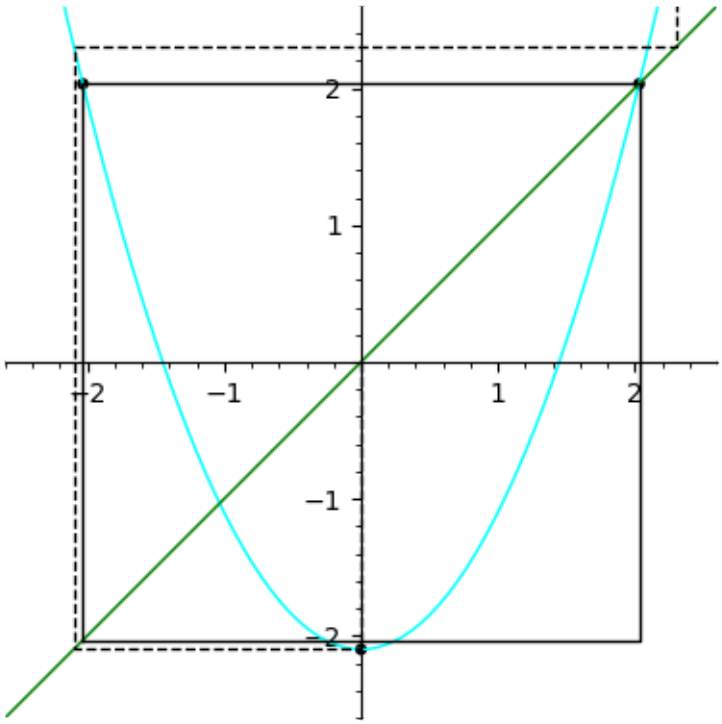
[7]:
```
c5 = -3
b5 = (1/2 + sqrt(- c5 + 1/4)).n()
s5 = 8
pf5 = plot(f(c = c5), (x, -s5, s5), color = 'cyan')
pd5 = plot(d, (x, -s5, s5), color = 'green')
p5 = list_plot([(0, c5)], color = 'black', size = 20)
it = 0
orbit = [(0,0)]
for i in range(3):
    nit = f(c = c5, x = it)
    orbit.append([it,nit])
    orbit.append([nit,nit])
    it = nit
porbit = list_plot(orbit, plotjoined = True, linestyle = '--', color = 'black')
(pf5 + pd5 + p5 + porbit).show(aspect_ratio = 1, xmin = -4, xmax = 8, ymin =␣
↪-5, ymax = 8)
```



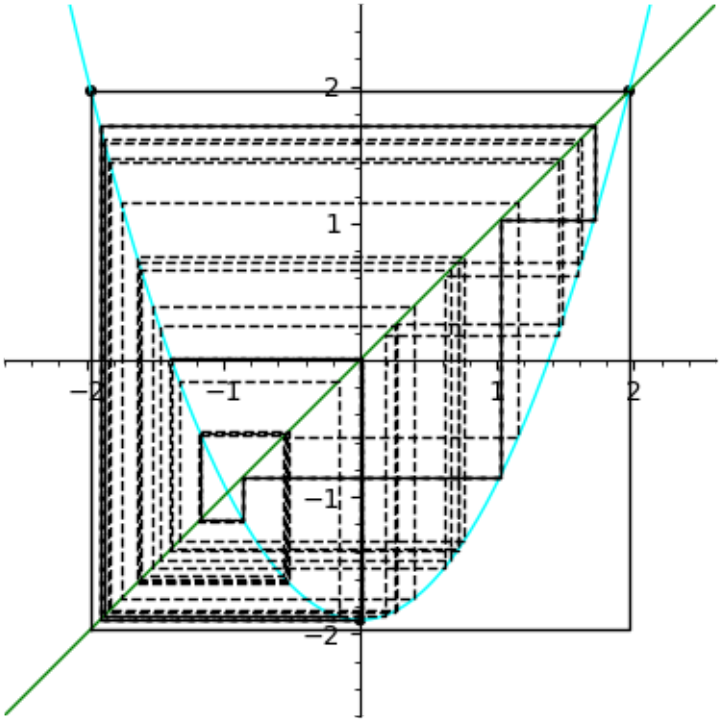
[8]:
```
c5 = -2.1
b5 = (1/2 + sqrt(- c5 + 1/4)).n()
s5 = 8
pf5 = plot(f(c = c5), (x, -s5, s5), color = 'cyan')
pd5 = plot(d, (x, -s5, s5), color = 'green')
p5 = list_plot([(0, c5)], color = 'black', size = 20)
it = 0
orbit = [(0,0)]
for i in range(6):
    nit = f(c = c5, x = it)
    orbit.append([it,nit])
    orbit.append([nit,nit])
    it = nit
porbit = list_plot(orbit, plotjoined = True, linestyle = '--', color = 'black')
(pf5 + pd5 + p5 + porbit).show(aspect_ratio = 1, xmin = -4, xmax = 5, ymin =␣
 ↪-5, ymax = 8)
```



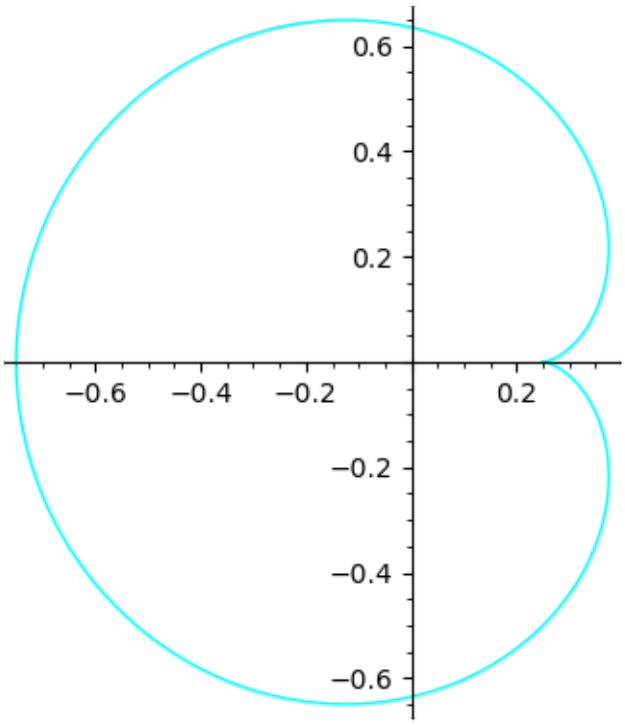
[9]:
```
c5 = -2.1
b5 = (1/2 + sqrt(- c5 + 1/4)).n()
s5 = 8
pf5 = plot(f(c = c5), (x, -s5, s5), color = 'cyan')
pd5 = plot(d, (x, -s5, s5), color = 'green')
p5 = list_plot([(0, c5), (-b5, b5), (b5, b5)], color = 'black', size = 20)
sq5 = list_plot([(-b5, -b5),(-b5, b5),(b5, b5),(b5, -b5),(-b5, -b5)],␣
 ↪plotjoined = True, color = 'black')
it = 0
orbit = [(0,0)]
for i in range(10):
    nit = f(c = c5, x = it)
    orbit.append([it,nit])
    orbit.append([nit,nit])
    it = nit
porbit = list_plot(orbit, plotjoined = True, linestyle = '--', color = 'black')
(pf5 + pd5 + p5 + sq5 + porbit).show(aspect_ratio = 1, xmin = -2.5, xmax = 2.5,␣
 ↪ymin = -2.5, ymax = 2.5)
```



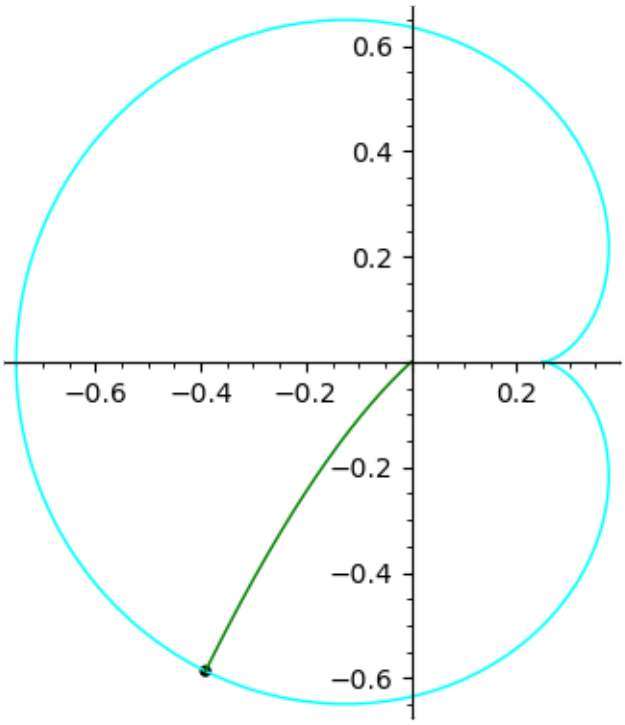
[10]:
```
c5 = -1.9
b5 = (1/2 + sqrt(- c5 + 1/4)).n()
s5 = 8
pf5 = plot(f(c = c5), (x, -s5, s5), color = 'cyan')
pd5 = plot(d, (x, -s5, s5), color = 'green')
p5 = list_plot([(0, c5), (-b5, b5), (b5, b5)], color = 'black', size = 20)
sq5 = list_plot([(-b5, -b5),(-b5, b5),(b5, b5),(b5, -b5),(-b5, -b5)],␣
↪plotjoined = True, color = 'black')
it = 0
orbit = [(0,0)]
for i in range(50):
    nit = f(c = c5, x = it)
    orbit.append([it,nit])
    orbit.append([nit,nit])
    it = nit
porbit = list_plot(orbit, plotjoined = True, linestyle = '--', color = 'black')
(pf5 + pd5 + p5 + sq5 + porbit).show(aspect_ratio = 1, xmin = -2.5, xmax = 2.5,␣
↪ymin = -2.5, ymax = 2.5)
```



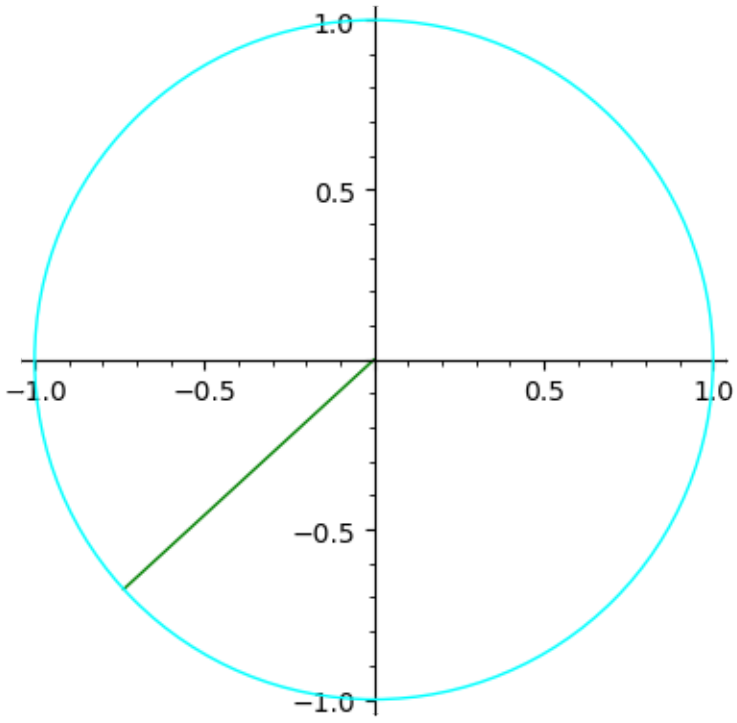
[11]:
```
# Main cardioid
var('t, r')
cx = r*(1/2)*cos(2*pi*t) - r^2*(1/4)*cos(4*pi*t)
cy = r*(1/2)*sin(2*pi*t) - r^2*(1/4)*sin(4*pi*t)
cardioid = parametric_plot((cx(r = 1), cy(r = 1)), (t, 0, 1), color = 'cyan')
cardioid.show(aspect_ratio = 1)
```



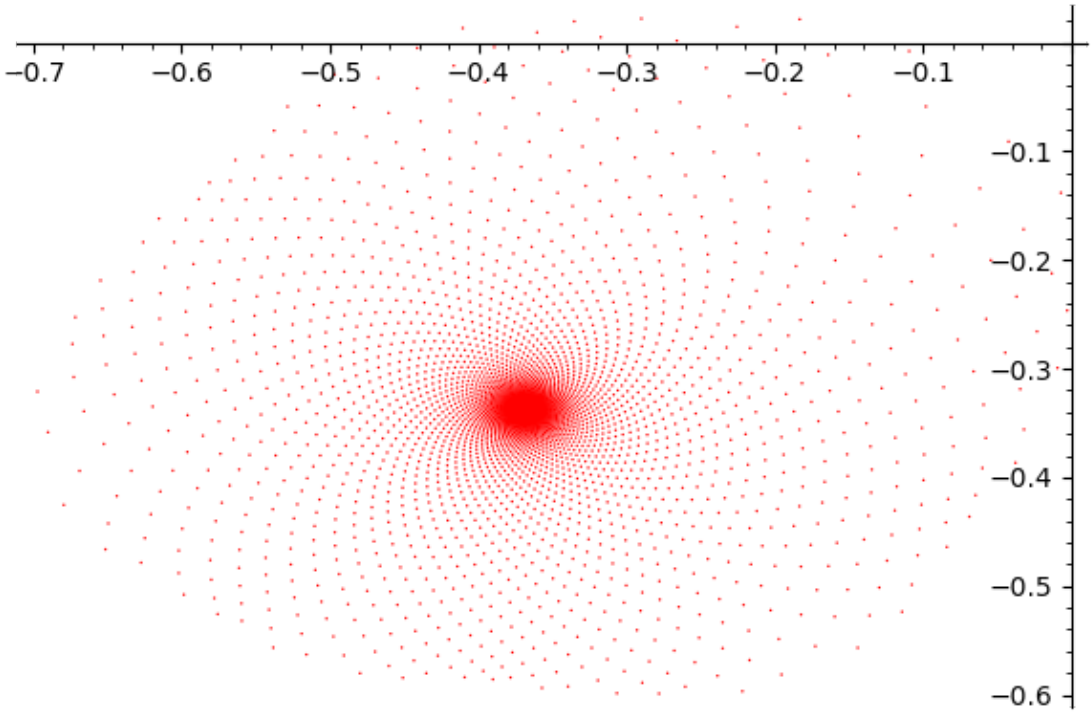
[12]:
```
# Internal rays of the main cardioid
varphi = ((1 + sqrt(5))/2).n()
internal_ray = parametric_plot((cx(t = varphi),cy(t = varphi)), (r, 0, 1),␣
 ↪color = 'green')
pvarphi = list_plot([(cx(t = varphi, r = 1), cy(t = varphi, r = 1))], size =␣
 ↪20, color = 'black')
(cardioid + internal_ray + pvarphi).show(aspect_ratio = 1)
```



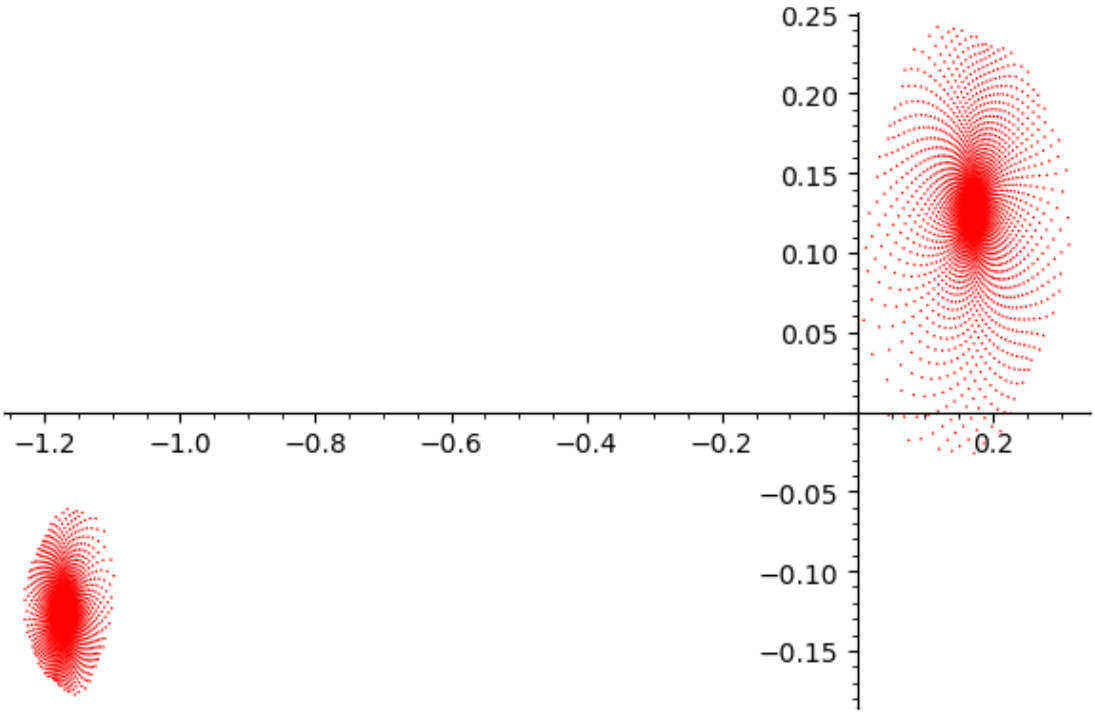
[13]:
```
# From the corresponding ray in the unit disk
cir = circle((0,0), 1, color = 'cyan')
ray = list_plot([(0,0), (cos(2*pi*varphi),sin(2*pi*varphi))], plotjoined =␣
 ↪True, color = 'green')
(cir + ray).show()
```



[14]:
```
# Critical orbit of parameters in the internal ray
r = 0.999
l = r*exp(2*pi*I*varphi)
c0 = (l/2 - l^2/4).n()
z = 0
C0 = []
for i in range(50000):
    z = z^2 + c0
    C0.append(z)
complex_orbit = list_plot(C0,pointsize=1,rgbcolor=(1,0,0))
complex_orbit.show()
```



[15]:
```
# Hyperbolic component of period 2
# Critical orbit of parameters in the internal ray
r = 0.999
l = r*exp(2*pi*I*varphi)
c0 = (l/4 - 1).n()
z = 0
CO = []
for i in range(100000):
    z = z^2 + c0
    CO.append(z)
complex_orbit = list_plot(CO,pointsize=1,color='red')
complex_orbit.show()
```


0.25
0.20
0.15
0.10
0.05
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0.2
−0.05
−0.10
−0.15