MTH 161
Final Exam
Sunday, December 17, 2017

Last Name (Family Name) ________________________________________________
First Name (Given Name) _______________________________________________
Student ID Number: _____________________________________________________
Circle your instructor and class time:

Hambrook (MW 10:25)    Hambrook (MW 2:00)
Lorman (TuTh 9:40)    Lubkin (MW 9:00)    Xi (TuTh 3:25)

Please read the following instructions very carefully:

• Only pens and pencils are allowed. The presence of notes, calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
• This exam has 15 problems and 24 pages. Check that your exam is complete when you start.
• Show your work. You may not receive full credit if insufficient justification is given.
• Clearly circle or label your final answers.
• If you need extra space, use the back of the opposite page, and write that you are doing so.
• Sign the following academic honesty statement: I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ____________________________

<table>
<thead>
<tr>
<th>Part A</th>
<th></th>
<th>Part B</th>
</tr>
</thead>
<tbody>
<tr>
<td>QUESTION</td>
<td>VALUE</td>
<td>SCORE</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Part A
1. (20 points)

(a) If \( f(x) = \frac{x + 1}{2x + 1} \), find a formula for \( f^{-1}(x) \).

(b) Solve \( \ln x + \ln(x - 1) = \ln 6. \)
(c) Find the exact value of \( \tan^{-1}\left(\frac{-1}{9}\right) \).

(d) Solve \(|x| + |2x - 1| \geq 7\).
2. (16 points) Compute the derivative (with respect to $x$) of each of the following functions.

(a) \( \frac{x^3}{\cos(x^3)} \)

(b) \( e^{\sqrt{\ln x}} \)
(c) $\sqrt{1 + x^2} + \frac{1}{\sqrt{1 + x^3}}$

(d) $(\ln x)^{\ln x}$
3. **(16 points)** Evaluate the following limits.

(a) \( \lim_{x \to 1^+} \frac{x^2 - 9}{x^2 + 2x - 3} \)

(b) \( \lim_{x \to -\infty} \frac{\sqrt{x^6 + 1}}{(2x + 1)^3} \)
(c) $\lim_{x \to 0} \frac{\sqrt{ax + b^2} - b}{x}$ (where $a$ and $b$ are positive constants)

(d) $\lim_{x \to 0} \frac{|2x - 1| - |2x + 1|}{x}$
4. (12 points) Consider the curve $y \sin 2x = x \cos 2y$.

(a) Find $\frac{dy}{dx}$ at the point $(\pi/2, \pi/4)$.

(b) Find an equation for the tangent line to the curve at the point $(\pi/2, \pi/4)$. 

5. (14 points) A plane is climbing at an angle of 30° while flying at a constant speed of 300 km/h. It passes over a ground radar station at an altitude of 7 km. At what rate is the distance from the plane to the radar station increasing 1 minute later?
6. **(14 points)** The height of the foam in a glass of (root) beer decreases at a rate proportional to the current height. The glass is filled with (root) beer so that the top 5 cm is foam. After 60 seconds, only 2 cm of foam remains.

(a) Find an expression for the height of the foam \( t \) seconds after the (root) beer is poured.

(b) At what time is the height of the foam 4 cm?

(c) How long must we wait until the foam completely disappears?
7. (8 points) For each statement below, circle T (true) if the statement is always true. Otherwise, circle F (false)

(a) T or F If \( f \) is continuous on \((a, b)\) and \( f(a) < 0 < f(b) \), then there is a number \( c \) in \((a, b)\) such that \( f(c) = 0 \).

(b) T or F If \( f'(0) = 5 \), then \( \lim_{h \to 0} \frac{f(h)}{h} = 5 \).

(c) T or F If \( f(1) = g(1) \) and \( f'(x) \leq g'(x) \) for all \( x \) in \([0, 1]\), then \( f(0) \geq g(0) \).

(d) T or F \( f(x) = x|x| \) is differentiable at every real number \( x \).
Part B
8. (8 points) For each statement below, circle T (true) if the statement is always true. Otherwise, circle F (false)

(a) T or F If $f$ is continuous, then $\int_0^5 f(x)dx \leq \int_0^5 |f(x)|dx$

(b) T or F If $f$ is continuous, then $\int_0^5 f(x)dx \leq \int_0^5 (f(x))^2dx$

(c) T or F If $f$ is continuous, then $\int_0^5 f(x)dx \leq \int_0^5 (f(x) + 2)dx$

(d) T or F If $f$ is continuous, then $\int_0^5 f(x)dx \leq \int_0^{10} f(x)dx$
9. (12 points)

(a) Let \( f(x) \) be an increasing function. If a Riemann sum with right endpoints is used to approximate \( \int_0^1 f(x) \, dx \), must the Riemann sum be larger than the integral? Justify your answer with an appropriate sketch.

(b) Let \( f(x) \) be an increasing function and let \( L(x) = f'(1)(x - 1) + f(1) \) be the linear approximation function of \( f(x) \) at 1. Must \( L(1.01) \) be larger than \( f(1.01) \)? Justify your answer with an appropriate sketch.
10. (8 points) Evaluate the following limits.

(a) \( \lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1-4x}}{x} \)

(b) \( \lim_{x \to 0^+} (\tan 2x)^x \)
11. (15 points) Evaluate the following integrals.

(a) \[ \int_1^{e^2} \frac{\sqrt{\ln x + 1}}{x} \, dx \]

(b) \[ \int \frac{x^5}{\sqrt{x^4 + 1}} \, dx \]
(c) $\int_{0}^{3} |e^x - 2| \, dx$
12. (20 points) Consider the function $f$ with its first and second derivatives:

$$f(x) = \frac{1}{\sqrt[3]{x^3} + 1}, \quad f'(x) = \frac{-x^2}{(\sqrt[3]{x^3} + 1)^2}, \quad f''(x) = \frac{2x(x^3 - 1)}{(\sqrt[3]{x^3} + 1)^3}.$$

(a) Find the domain of $f(x)$.

(b) List all $x$-intercepts and $y$-intercepts of $f(x)$.
Reminder: $f(x) = \frac{1}{\sqrt[3]{x^3} + 1}$, $f'(x) = \frac{-x^2}{(\sqrt[3]{x^3} + 1)^4}$, $f''(x) = \frac{2x(x^3 - 1)}{(\sqrt[3]{x^3} + 1)^7}$.

(c) Compute $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ for any vertical asymptotes $x = a$.

(d) Compute $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$. List all horizontal asymptotes of $f(x)$. 
Reminder: $f(x) = \frac{1}{\sqrt[3]{x^3 + 1}}, \quad \frac{\text{d}f}{\text{d}x}(x) = \frac{-x^2}{(\sqrt[3]{x^3 + 1})^4}, \quad \frac{\text{d}^2f}{\text{d}x^2}(x) = \frac{2x(x^3 - 1)}{(\sqrt[3]{x^3 + 1})^7}$.

(e) On what intervals is $f(x)$ increasing? decreasing?

(f) On what intervals is $f(x)$ concave up? concave down?
13. **(10 points)** Sketch the graph of a function $f(x)$ that satisfies the following properties:

- $x$-intercepts: $-3, 3$
- $y$-intercepts: $2$
- vertical asymptotes: $x = -5$ and $x = 5$
- horizontal asymptotes: $y = -1$ and $y = 1$
- $f'(x) > 0$ on $(-\infty, -5) \cup (-5, 0) \cup (5, \infty)$
- $f'(x) < 0$ on $(0, 5)$
- $f''(x) > 0$ on $(-\infty, -5)$
- $f''(x) < 0$ on $(-5, 5) \cup (5, \infty)$
14. (12 points)

(a) A particle is moving with the given velocity and position data. Find the position function $s(t)$ of the particle.

$$v(t) = 10 \sin t + 3 \cos t, \quad s(\pi/4) = 12$$

(b) Let $f(x) = \int_1^{x^2} (t - 4) e^{-t^2} dt$ for all real numbers $x$. On what intervals is $f(x)$ an increasing function?
15. **(15 points)** A woman at a point $A$ on the shore of a circular lake with radius 2 miles wants to arrive at the point $C$ diametrically opposite $A$ on the other side of the lake in the shortest possible time (see the figure). She can walk at the rate of 4 miles/h and row a boat at 2 miles/h. How should she proceed? Justify your answer. (It may help to know that $\sqrt{3} = 1.73\ldots$)
Formula Sheet
You may tear this page off.

\[
\begin{align*}
\sin(x + y) &= \sin x \cos y + \cos x \sin y \\
\sin(x - y) &= \sin x \cos y - \cos x \sin y \\
\cos(x + y) &= \cos x \cos y - \sin x \sin y \\
\cos(x - y) &= \cos x \cos y + \sin x \sin y
\end{align*}
\]

\[
\begin{align*}
\sin 2x &= 2 \sin x \cos x \\
\cos 2x &= \cos^2 x - \sin^2 x \\
&= 2 \cos^2 x - 1 \\
&= 1 - 2 \sin^2 x \\
\cos^2 x &= \frac{1 + \cos 2x}{2} \\
\sin^2 x &= \frac{1 - \cos 2x}{2}
\end{align*}
\]
Scratch Paper. You may tear this page off. Nothing you write on this page will be graded.