Math 161: Calculus IA
Final Exam
December 15, 2009

NAME (please print legibly): ________________________________
Your University ID Number: ________________________________

Indicate your instructor with a check in the box:

<table>
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<th>Instructor</th>
<th>Time</th>
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<tr>
<td>Nsoki Mavinga</td>
<td>MWF 11:00 - 11:50 AM</td>
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<tr>
<td>Nick Rogers</td>
<td>MWF 10:00 - 10:50 AM</td>
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<tr>
<td>Ibrahim Unal</td>
<td>MWF 9:00 - 9:50 AM</td>
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<tr>
<td>Troy Winfree</td>
<td>TTh 2:00 - 3:15 PM</td>
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- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- Clearly circle or label your final answers.

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Part A

1. (8 points) Let \( f(x) = \ln(x + 6) \).

(a) Find a formula for \( f^{-1}(x) \), the inverse of \( f \).

(b) Find the domain and range of \( f^{-1} \).
2. (8 points) The graph of a function $f(x)$ is shown below.

For each of the following functions, identify the corresponding graph (1 – 4) in the box.

(a) $f(-x) + 2$  
(b) $f(x + 3) - 2$

(c) $f(2x + 1)$  
(d) $-f(x) + 1$
3. (10 points) Let \( c \) be a constant, and consider

\[
f(x) = \begin{cases} 
  cx^5 - x & \text{if } x \leq 1 \\
  x^2 - cx & \text{if } x > 1 
\end{cases}
\]

(a) Find \( \lim_{x \to 1^-} f(x) \).

(b) Find \( \lim_{x \to 1^+} f(x) \).

(c) For what value of \( c \) is \( f(x) \) continuous at \( x = 1 \)?
4. (12 points) Let $f$ and $g$ be differentiable functions. The following table gives the values of $f$, $f'$, $g$ and $g'$ at $x = 0, 1, 2$.

<table>
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<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$f'(x)$</th>
<th>$g'(x)$</th>
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<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>$\sqrt{2}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>$-1$</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1/3</td>
<td>$e$</td>
<td>$\pi$</td>
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(a) Let $H(x) = (x - 3)^3 f(x)$. Compute $H'(2)$.

(b) Let $G(x) = f(g(x) + 2x)$. Compute $G'(1)$.

(c) Let $F(x) = \frac{g(x)}{f(x) + x}$. Compute $F'(0)$.
5. **(12 points)** Find each of the following derivatives.

(a) $f'(x)$, where $f(x) = (7 - \sqrt{x})e^{x^2}$

(b) $g'(t)$, where $g(t) = \tan(\sqrt{t^2} + 1)$

(c) $h'(x)$, where $h(x) = \frac{1 + \sin x}{2^x - x^2}$

(d) $\frac{dy}{ds}$, where $y = \ln(\ln s)$
Part B

6. (8 points) A 15-foot ladder rests against a wall. The foot of the ladder begins to slide away from the wall at a rate of 2 feet per second. At what rate is the top of the ladder sliding down the wall when the foot of the ladder is 9 feet away from the wall?
7. (14 points) Evaluate the following limits.

(a) \[ \lim_{x \to 3} \frac{x^2 - 5x + 6}{x^2 - 2x - 3} \]

(b) \[ \lim_{x \to 0} \frac{x^2 + \sin(2x)}{x} \]

(c) \[ \lim_{x \to \infty} x^2 e^{-x} \]
(d) \( \lim_{x \to 0^+} x^x \)

(e) \( \lim_{x \to \infty} (\sqrt{x^2 + 2} - x) \)

(f) \( \lim_{x \to 5} g(x) \), where \( 10x - 20 \leq g(x) \leq x^2 + 5 \) for all \( x \).
8. (10 points) Find the absolute maximum and absolute minimum of

\[ f(x) = e^x \cos x \]

on the closed interval \([-\pi/2, \pi/2]\).
9. **(10 points)** Suppose $C$ is the curve defined by the equation

$$\ln(xy) = (x - y)^2.$$ 

This equation defines $y$ implicitly as a function of $x$.

(a) Find a formula for $\frac{dy}{dx}$ in terms of $x$ and $y$.

(b) Find an equation for the tangent line to the curve $C$ at the point $(1, 1)$. 
10. (8 points) Find the derivative \( \frac{dy}{dx} \) of each of the following functions.

(a) \( y = \frac{(x^2 + 1)^3 \sin x}{x^2(1 + e^x)^3} \)

(b) \( y = (1 + \ln x)^2 \)
11. (20 points) Let \( f(x) = \frac{x^2 - 3x}{(x-1)^2} \). It can be shown that
\[
f'(x) = \frac{x + 3}{(x-1)^3} \quad \text{and} \quad f''(x) = \frac{-2x - 10}{(x-1)^4}.
\]

(a) Find the \( x \)- and \( y \)-intercepts of \( f(x) \).

(b) Find the horizontal and vertical asymptote(s) of \( f(x) \).

Continued on next page
(c) Find the point(s) at which $f(x)$ has a local maximum and the point(s) at which $f(x)$ has a local minimum.

(d) Find the interval(s) on which $f(x)$ is increasing and the interval(s) on which it is decreasing.

(e) Find the inflection point(s) of $f(x)$.

Continued on next page
(f) Find the interval(s) on which \( f(x) \) is concave up and the interval(s) on which it is concave down.

(g) Sketch the graph of \( f(x) \). Please label all intercepts, local maxima and minima, and inflection points.
12. **(10 points)** Ibrahim wants to build a rectangular pen with three parallel partitions, as shown below. He has 400 feet of fencing. What dimensions of the rectangular pen maximize the total area?
13. (10 points) Suppose an object moves in a straight line with velocity (in meters per second) at time $t$ (in seconds) given by

$$v(t) = t^2 - 3t + 2.$$ 

The initial position of the object is 3 meters to the right of the origin.

(a) Compute the acceleration of the object after 5 seconds.

(b) Find an equation for the position as a function of $t$.

(c) Find the total distance traveled during the time interval $[1, 3]$. 
14. (8 points) Consider \( f(x) = \sqrt{x} \) on the interval \([0, 4]\). Divide \([0, 4]\) into 4 subintervals of equal length.

(a) Using the right endpoints of the four subintervals, write down a Riemann Sum to approximate \( \int_0^4 \sqrt{x} \, dx \). You do not need to evaluate this sum numerically, but please do not leave your answer in sigma notation.

(b) Is your answer to part (a) larger or smaller than \( \int_0^4 \sqrt{x} \, dx \)? Explain.
15. (12 points) Let

\[ f(x) = \begin{cases} 
1 + \sqrt{9 - x^2} & \text{if } -3 \leq x \leq 3; \\
-x + 4 & \text{if } 3 < x \leq 6.
\end{cases} \]

Evaluate the following integrals by interpreting them in terms of areas. *Hint:* the graph of \( f(x) \) is shown in problem 2.

(a) \( \int_{-3}^{3} f(x) \, dx \)

(b) \( \int_{3}^{6} f(x) \, dx \)

(c) \( \int_{-3}^{6} f(x) \, dx \)
16. (12 points) Let $f(x)$ be a continuous function such that
\[
\int_1^3 f(x) \, dx = 3 \quad \text{and} \quad \int_1^2 f(x) \, dx = -2.
\]

(a) Find $\int_1^3 (3f(x) + 5) \, dx$.

(b) Find $\int_3^2 f(x) \, dx$.

(c) Suppose that $g(x)$ is a continuous function such that $\sqrt{2} \leq g(x) \leq 2$ on the closed interval $[3, 10]$. Find upper and lower bounds for $\int_3^{10} g(x) \, dx$. 
17. (12 points)

(a) If \( f(x) = \int_{1}^{3x} \frac{t^2 - 9}{2 + t^4} \, dt \), find \( f'(x) \).

(b) At what value(s) of \( x \) does the function \( f(x) \) from part (a) have a local minimum?

(c) Suppose \( g(1) = 7 \), \( g'(x) \) is continuous, and \( \int_{1}^{5} g'(x) \, dx = 11 \). Find \( g(5) \).
18. (16 points) Evaluate the following definite and indefinite integrals.

(a) \[ \int_1^2 \left(2t + \frac{3}{t^2}\right) \, dt. \]

(b) \[ \int_0^\pi (3\cos x - 2x + 5) \, dx. \]

(c) \[ \int \frac{2t + 3t^2}{\sqrt{t}} \, dt. \]

(d) \[ \int \left(3e^x - \frac{2}{x^2 + 1}\right) \, dx. \]