Math 161: Calculus IA
Midterm Exam 2
November 19, 2009

NAME (please print legibly): ________________________________
Your University ID Number: ________________________________
Indicate your instructor with a check in the box:

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Time</th>
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<tbody>
<tr>
<td>Nsoki Mavinga</td>
<td>MWF 11:00 - 11:50 AM</td>
</tr>
<tr>
<td>Nick Rogers</td>
<td>MWF 10:00 - 10:50 AM</td>
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<tr>
<td>Ibrahim Unal</td>
<td>MWF 9:00 - 9:50 AM</td>
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<tr>
<td>Troy Winfree</td>
<td>TTh 2:00 - 3:15 AM</td>
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- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Clearly circle or label your final answers.

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<th>QUESTION</th>
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1. (12 points) Consider the curve \( C \) defined by \( x^2y + y^4 = 2x + 4 \). Assume that this equation defines \( y \) implicitly as a function of \( x \).

(a) Find an expression for \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).

(b) Find an equation for the line tangent to \( C \) at the point \((-1, 1)\).
2. (15 points) Find the derivative \( \frac{dy}{dx} \) of each of the following functions.

(a) \( y = x + \ln |\cos x| \)

(b) \( y = \frac{x^2(1 + e^x)}{x^3 \sqrt{x} + 1} \)

(c) \( y = (1 + \sqrt{x})^{x^2} \)
3. (10 points) Consider the function

\[ f(x) = \sqrt[3]{x} + 2. \]

(a) Find the linear approximation to \( f(x) \) at \( x = 6 \).

(b) Estimate \( \sqrt[3]{8.05} \) using linear approximation.
Ship A has left port and is moving west at 6 mph, and Ship B is approaching port from the south at 7 mph. Ship A is 15 miles from port, and Ship B is 8 miles from port. Are the ships getting closer together or farther apart? At what speed?
5. (16 points) Evaluate the following limits. If it does not exist write DNE. If it is infinite, determine whether it is $+\infty$ or $-\infty$. Explain your reasoning.

(a) \[ \lim_{x \to 1} \frac{2x - 2}{x - 1} \]

(b) \[ \lim_{x \to \pi} \frac{\sin x}{e^x + 1} \]

(c) \[ \lim_{x \to 1/2} (2x - 1) \tan(\pi x) \]

(d) \[ \lim_{x \to \infty} x^{1/x} \]
6. (12 points) Consider the function \( f(x) = x^3 - 12x \) on the interval \([0, 3]\).

(a) Find all critical numbers of \( f(x) \) in the given interval.

(b) Find the absolute maximum and minimum values of \( f(x) \) on \([0, 3]\).
7. (15 points) Consider the function \( f(x) = 3x^2 - x^3 - 1 \).

(a) Find the interval(s) on which \( f(x) \) is increasing, and the interval(s) on which \( f(x) \) is decreasing.

(b) Find all points where \( f(x) \) has a local maximum or minimum. (Be sure to give the \( x \)- and \( y \)-coordinates of each point.)

(c) Find the interval(s) on which \( f(x) \) is concave up, and the interval(s) on which it is concave down.

Continued on next page
(d) Find the inflection point(s). (Be sure to give the x- and y-coordinates.)

(e) Use the information from parts (a)-(d) to sketch the graph of \( f(x) \).
8. (10 points) Let \( f(x) \) be the function
\[
f(x) = x - 2 \sin x
\]
with domain \([0, \pi]\).

(a) Compute the average slope of \( f(x) \) on \([0, \pi]\).

(b) Complete the following statement of the Mean Value Theorem for the function \( f(x) \):

Since \( f(x) \) is continuous on \([0, \pi]\) and differentiable on \((0, \pi)\), there exists a number \( c \) in the open interval \((0, \pi)\) such that

(c) Find the number \( c \) whose existence is guaranteed by the Mean Value Theorem for \( f(x) \) on \([0, \pi]\).